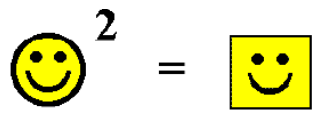
# Welcome (Back) to IB Math - Standard Level Year 2



Some things to know:

1. Lots of info at www.aleimath.blogspot.com
2. HW - yup. You know you love it! Be prepared to present.
3. Content:

| Topic | | Hrs | Notes |
|---|---|---|---|
| Topic 1 | Algebra | 8 | Covered in Year 1 |
| Topic 2 | Functions and equations | 24 | Covered in Year 1 |
| Topic 3 | Circular functions and trigonometry | 16 | To be studied Year 2 |
| Topic 4 | Matrices | 10 | Covered in Year 1 |
| Topic 5 | Vectors | 16 | Covered in Year 1 |
| Topic 6 | Statistics and probability | 30 | To be studied Year 2 |
| Topic 7 | Calculus | 36 | To be studied Year 2 |
| | Portfolio (Internal Assessment or IA) | 10 | 2nd IA done Year 2 |
| | Total | **150** | |

4. Grading - Ultimately, you need to pass the IB exam!
   40% Tests
   20% Quizzes
   30% Presentations (HW!)
   10% Classwork
5. Bring: Notebooks ($3!), pencil(s), **and you**!

## Topic 3—Circular functions and trigonometry 16 hrs

### Aims

The aims of this section are to explore the circular functions and to solve triangles using trigonometry.

### Details

| | Content | Amplifications/inclusions | Exclusions |
|---|---|---|---|
| **3.1** | The circle: radian measure of angles; length of an arc; area of a sector. | Radian measure may be expressed as multiples of $\pi$, or decimals. | |
| **3.2** | Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.<br>Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$.<br>The identity $\cos^2\theta+\sin^2\theta=1$. | Given $\sin\theta$, finding possible values of $\cos\theta$ without finding $\theta$.<br>Lines through the origin can be expressed as $y=x\tan\theta$, with gradient $\tan\theta$. | The reciprocal trigonometric functions $\sec\theta$, $\csc\theta$ and $\cot\theta$. |
| **3.3** | Double angle formulae: $\sin 2\theta=2\sin\theta\cos\theta$; $\cos 2\theta=\cos^2\theta-\sin^2\theta$. | Double angle formulae can be established by simple geometrical diagrams and/or by use of a GDC. | Compound angle formulae. |
| **3.4** | The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; their periodic nature; and their graphs.<br>Composite functions of the form $f(x)=a\sin(b(x+c))+d$. | On examination papers: radian measure should be assumed unless otherwise indicated by, for example, $x\mapsto\sin x^\circ$.<br>Example: $f(x)=2\cos(3(x-4))+1$.<br>Examples of applications: height of tide, Ferris wheel. | The inverse trigonometric functions: $\arcsin x$, $\arccos x$ and $\arctan x$. |
| **3.5** | Solution of trigonometric equations in a finite interval.<br>Equations of the type $a\sin(b(x+c))=k$.<br>Equations leading to quadratic equations in, for example, $\sin x$.<br>Graphical interpretation of the above. | Examples:<br>$2\sin x=3\cos x$, $0\le x\le 2\pi$.<br>$2\sin 2x=3\cos x$, $0^\circ\le x\le 180^\circ$.<br>$2\sin x=\cos 2x$, $-\pi\le x\le\pi$.<br>Both analytical and graphical methods required. | The general solution of trigonometric equations. |
| **3.6** | Solution of triangles.<br>The cosine rule: $c^2=a^2+b^2-2ab\cos C$.<br>The sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.<br>Area of a triangle as $\frac{1}{2}ab\sin C$. | Appreciation of Pythagoras' theorem as a special case of the cosine rule.<br>The ambiguous case of the sine rule.<br>Applications to problems in real-life situations, such as navigation. | |

Unit plan on website has HW assignments for these chapters.

## Chapter 8

## The unit circle and radian measure

- **A** Radian measure
- **B** Arc length and sector area
- **C** The unit circle and the basic trigonometric ratios
- **D** The equation of a straight line

## Chapter 9

## Non-right angled triangle trigonometry

- **A** Areas of triangles
- **B** The cosine rule
- **C** The sine rule
- **D** Using the sine and cosine rules

## Chapter 10

## Advanced trigonometry

- **A** Observing periodic behaviour
- **B** The sine function
- **C** Modelling using sine functions
- **D** The cosine function
- **E** The tangent function
- **F** General trigonometric functions
- **G** Trigonometric equations
- **H** Using trigonometric models
- **I** Trigonometric relationships
- **J** Double angle formulae
- **K** Trigonometric equations in quadratic form

Right Angle Trig - Review

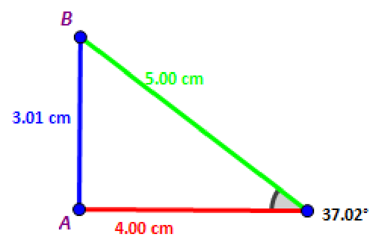
GSP Sketch to Explore

m∠BCA = 37.02°

Hypotenuse = 5.00 cm

Opposite = 3.01 cm

Adjacent = 4.00 cm




Hide Sin

$$\frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{3.01\text{ cm}}{5.00\text{ cm}} = 0.60 = \text{Sin}(37.02°)$$

Hide Cosine

$$\frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{4.00\text{ cm}}{5.00\text{ cm}} = 0.80 = \text{Cos}(37.02°)$$

Hide Tangent

$$\frac{\text{Opposite}}{\text{Adjacent}} = \frac{3.01\text{ cm}}{4.00\text{ cm}} = 0.75 = \text{Tan}(37.02°)$$

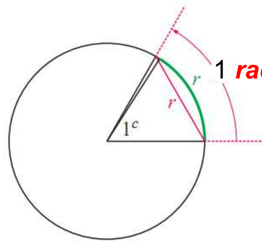
Mnemonic

SOH CAH TOA
Some Old Hippie Caught Another Hippie Tripping on Acid

## A RADIAN MEASURE

Lots of theories about where 360 degrees came from. The Babylonians has a base 60 number system that may have contributed. One version of a Mayan calendar used 20 cycles of 18 days (360) plus 5 unlucky days! The Persian calendar used 360 days for a year. Note the connection to hours, minutes and seconds - time was initially measured based on astronomical cycles that were assumed to be circular.

Roger Cotes is generally credited with defining a new unit of measure. The ***radian***. It was natural to measure an angle by the length of the arc that it subtends. But that length would be different for different size circles. Unless, of course, you measure the length in "numbers of radii of the given circle". Thus the name "radians".




So how do we convert between the systems?

How many radians (radii) are there in a complete circle?

Distance around a circle is 2πr so:

360 degrees = 2π radians
or 180 degrees = π radians

Try a couple:

Convert $45^o$ to radians in terms of $\pi$.

$45^o = (45 \times \frac{\pi}{180})$ radians
$= \frac{\pi}{4}$ radians

*or*

$180^o = \pi$ radians
$\therefore \quad (\frac{180}{4})^o = \frac{\pi}{4}$ radians
$\therefore \quad 45^o = \frac{\pi}{4}$ radians

| | |
|---|---|
| Convert $\frac{5\pi}{6}$ to degrees. | $\frac{5\pi}{6}$<br>$= (\frac{5\pi}{6} \times \frac{180}{\pi})^o$<br>$= 150^o$ |

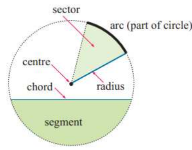
RightAngleTrigReview.PDF

Handout [Review set](Review set)
8A #1-4 odd cols, 5 all

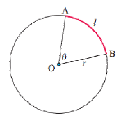
Homework
Discretion...

# B ARC LENGTH AND SECTOR AREA

First, some vocabulary




## ARC LENGTH



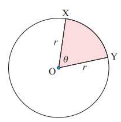

In the diagram, the **arc length** AB is $l$. $\theta$ is measured in **radians**.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\theta}{2\pi}$$

$$\therefore \frac{l}{2\pi r} = \frac{\theta}{2\pi}$$

$$\boldsymbol{l = \theta r}$$

## AREA OF SECTOR




In the diagram, the area of minor sector XOY is shaded. $\theta$ is measured in **radians**.

$$\frac{\text{area of minor sector XOY}}{\text{area of circle}} = \frac{\theta}{2\pi}$$

$$\therefore \frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\boldsymbol{A = \tfrac{1}{2}\theta r^2}$$

If $\theta$ is in **degrees**, $l = \frac{\theta}{360} \times 2\pi r$ and $A = \frac{\theta}{360} \times \pi r^2$.

A sector has radius 12 cm and angle 3 radians. Use radians to find its:

**a** arc length **b** area

**a** arc length $= \theta r$
$= 3 \times 12$
$= 36$ cm

**b** area $= \frac{1}{2}\theta r^2$
$= \frac{1}{2} \times 3 \times 12^2$
$= 216$ cm$^2$

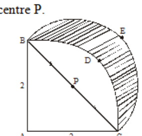
A sector has radius 8.2 cm and arc length 13.3 cm. Find the area of this sector.

$l = \theta r$ {$\theta$ in radians}

$\therefore \theta = \frac{l}{r} = \frac{13.3}{8.2}$

$\therefore$ area $= \frac{1}{2}\theta r^2$
$= \frac{1}{2} \times \frac{13.3}{8.2} \times 8.2^2$
$\approx 54.5$ cm$^2$

The diagram below shows a triangle and two arcs of circles.
The triangle ABC is a right-angled isosceles triangle, with AB = AC = 2. The point P is the midpoint of [BC].
The arc BDC is part of a circle with centre A.
The arc BEC is part of a circle with centre P.




(a) Calculate the area of the segment BDCP.
(b) Calculate the area of the shaded region BECD.

**(Total 6 marks)**

(a) area of sector ABDC $= \frac{1}{4}\pi(2)^2 = \pi$ (A1)

area of segment BDCP $= \pi -$ area of $\Delta$ABC (M1)
$= \pi - 2$ (A1) (C3)

(b) BP $= \sqrt{2}$ (A1)

area of semicircle of radius BP $= \frac{1}{2}\pi(\sqrt{2})^2 = \pi$ (A1)

area of shaded region $= \pi - (\pi - 2) = 2$ (A1)
(C3)

**[6]**

8B #1-4, 7-12 Arc length and sector area
QB: 3,4,10,13,17,20,33,37,45,55,57,69,74,77

## SL 2 Assignments: Week of 9/10/12

**Mon 9/10:**
8C.1: #1b,2a,3,4,5&6*,7bdf,8b,9 (Unit Circle & basic definitions)
8C.2: #1cd,2cd,3bd,4cd,5bd (Applications of unit circle)
QB: 3,4,10,13
**Wed 9/12:**
8C.3: #1-5 & 7 (Multiples of pi/4 & pi/6)
8D: #1&2 (Straight line & tangents)
QB: 17,20,33,37,45
**Fri 9/14:**
Review: Set 8B & 8C, p. 215-216 (Review)
QB: 55,57,69,74,77

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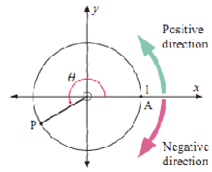
*(Coming up)*
**Mon 9/17:** Quiz 1 - Arc length, sector area, unit circle

Present 7-10,12,QB 3, 4

# C THE UNIT CIRCLE AND THE BASIC TRIGONOMETRIC RATIOS

A circle of radius one is one of the most fundamental geometric shapes. It is a very effective visual reference for many ideas involving trigonometry.

Unit Circle demo




From this we see some important principles:

$\cos\theta$ is the $x$-coordinate of P.
$\sin\theta$ is the $y$-coordinate of P.

These are more general definitions since they include negative values and angles greater than 90 deg.

$-1 \leqslant \cos\theta \leqslant 1$ and $-1 \leqslant \sin\theta \leqslant 1$ for all $\theta$.

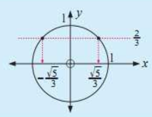
$\cos^2\theta + \sin^2\theta = 1.$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}.$ This is a more general definition of tangent.

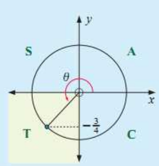
Try some applications:

Find the possible values of $\cos\theta$ for $\sin\theta = \frac{2}{3}$. Illustrate your answers.

$$\cos^2\theta + \sin^2\theta = 1$$
$$\therefore \quad \cos^2\theta + \left(\tfrac{2}{3}\right)^2 = 1$$
$$\therefore \quad \cos^2\theta = \tfrac{5}{9}$$
$$\therefore \quad \cos\theta = \pm\tfrac{\sqrt{5}}{3}$$



If $\sin\theta = -\frac{3}{4}$ and $\pi < \theta < \frac{3\pi}{2}$, find $\cos\theta$ and $\tan\theta$ without using a calculator.




Now $\cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \cos^2\theta + \frac{9}{16} = 1$

$\therefore \quad \cos^2\theta = \frac{7}{16}$

$\therefore \quad \cos\theta = \pm\frac{\sqrt{7}}{4}$

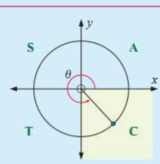
But $\pi < \theta < \frac{3\pi}{2}$, so $\theta$ is a quadrant 3 angle

$\therefore \quad \cos\theta$ is negative.

$\therefore \quad \cos\theta = -\frac{\sqrt{7}}{4}$

and $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = \dfrac{3}{\sqrt{7}}$

If $\tan\theta = -2$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin\theta$ and $\cos\theta$.




$\dfrac{\sin\theta}{\cos\theta} = -2$

$\therefore \quad \sin\theta = -2\cos\theta$

Now $\sin^2\theta + \cos^2\theta = 1$

$\therefore \quad 4\cos^2\theta + \cos^2\theta = 1$

$\therefore \quad 5\cos^2\theta = 1$

$\therefore \quad \cos\theta = \pm\frac{1}{\sqrt{5}}$

But $\frac{3\pi}{2} \leqslant \theta \leqslant 2\pi$, so $\theta$ is a quadrant 4 angle.

$\therefore \quad \cos\theta$ is positive and $\sin\theta$ is negative.

$\therefore \quad \cos\theta = \frac{1}{\sqrt{5}}$ and $\sin\theta = -\frac{2}{\sqrt{5}}$.

| | |
|---|---|
| 8C.1 | #1b,2a,3,4,5&6*,7bdf,8b,9 |
| 8C.2 | #1cd,2cd,3bd,4cd,5bd |
| QB | 3,10,13 |

*Use judgment on how many

Review of some important results from last time:

$\cos\theta$ is the $x$-coordinate of P.
$\sin\theta$ is the $y$-coordinate of P.

These are more general definitions since they include negative values and angles greater than 90 deg.

$-1 \leqslant \cos\theta \leqslant 1$ and $-1 \leqslant \sin\theta \leqslant 1$ for all $\theta$.

$\cos^2\theta + \sin^2\theta = 1.$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}.$ This is a more general definition of tangent.

Since there are $2\pi$ radians in a full revolution, if we add any integer multiple of $2\pi$ to $\theta$ then the position of P on the unit circle is unchanged.

So, for all $k \in \mathbb{Z}$ and angles $\theta$, $\cos(\theta + 2k\pi) = \cos\theta$ and $\sin(\theta + 2k\pi) = \sin\theta$.

Some results that you may have developed. **Don't** memorize. **Do** understand:

| | |
|---|---|
| sin(180 - θ) = sinθ | cos(180 - θ) = -cosθ |
| sin(90 - θ) = cosθ | cos(90 - θ) = sinθ |
| sin(-θ) = - sinθ | cos(-θ) = cosθ |

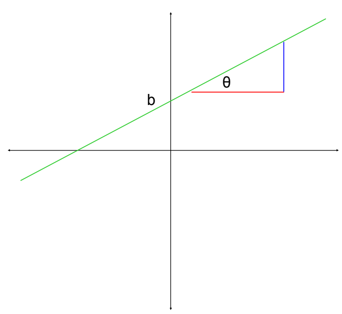
The most common angles are multiples of 30 or 45 degrees.
These correspond to multiples of π/6 and π/4 radians.

Because they are related to the common 30-60-90 and 45-45-90 triangles, the values of the sin and cos functions for these angles should be memorized.

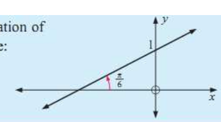
You need to be able to quickly construct a unit circle with exact coordinates, degree and radian measures of all angles that are multiples of 30 or 45 degrees.

- If $\theta$ is a **multiple of** $\frac{\pi}{2}$, the coordinates of the points on the unit circle involve 0 and $\pm 1$.
- If $\theta$ is a **multiple of** $\frac{\pi}{4}$, but not a multiple of $\frac{\pi}{2}$, the coordinates involve $\pm\frac{1}{\sqrt{2}}$.
- If $\theta$ is a **multiple of** $\frac{\pi}{6}$, but not a multiple of $\frac{\pi}{2}$, the coordinates involve $\pm\frac{1}{2}$ and $\pm\frac{\sqrt{3}}{2}$.

## D THE EQUATION OF A STRAIGHT LINE




Find the equation of the given line:




The line has gradient $m = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
and its $y$-intercept is 1.

$\therefore$ the line has equation $y = \frac{1}{\sqrt{3}}x + 1$.

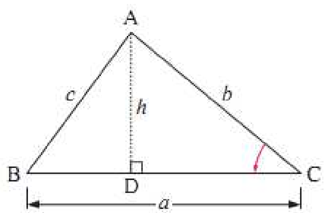
8C.3: #1-5 & 7 (Multiples of pi/4 & pi/6)
8D: #1&2 (Straight line & tangents)
QB: 20,27,33,37,45 (IB Practice)

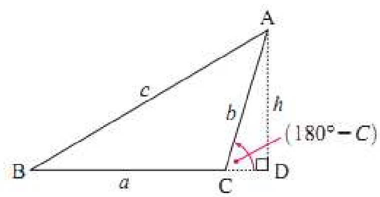
HW and classwork for Fri - review

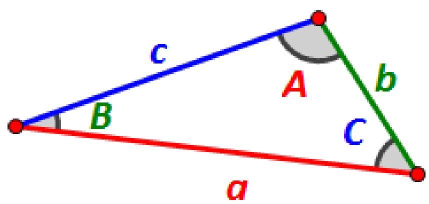
QB: 55,57,69,74,77 (IB Practice)
Review: Set 8B & 8C, p. 215-216 (Review)

# A AREAS OF TRIANGLES

Presented this on the fly. Develop by dropping an altitude from any side and finding the height from the sin of either of the two remaining angles. Works for obtuse since sin(180 - $\alpha$) = sin$\alpha$



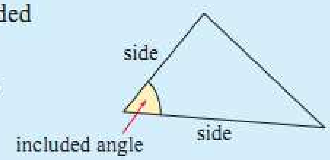




Area $\frac{1}{2}ab\sin C$

$A = \frac{1}{2}ab\sin C.$

Given the lengths of two sides of a triangle and the included angle between them, the area of the triangle is

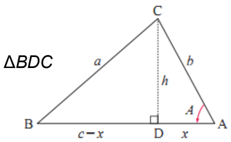
*a half of the product of two sides and the sine of the included angle.*




9A #1-11 Areas of Triangles

## B THE COSINE RULE

Consider the triangle at right:




Note that $h$ can be computed from $\Delta ADC$ or from $\Delta BDC$

Thus $b^2 - x^2 = a^2 - (c - x)^2$ (Pythagorus)

Let's simplify:

$b^2 - x^2 = a^2 - (c^2 - 2cx + x^2)$
$b^2 - x^2 + (c^2 - 2cx + x^2) = a^2$
$b^2 - x^2 + c^2 - 2cx + x^2 = a^2$
$b^2 + c^2 - 2cx = a^2$

But trig tells us that $x = b\cos A$ (Definition of cos)

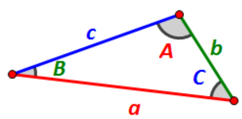
Substitute to get $b^2 + c^2 - 2bc\cos A = a^2$ The ***cosine rule***

**The Law of Cosines (Cosine Rule)**

For a triangle with sides of length *a*, *b*, & *c* whose opposite angles are given by *A*, *B*, & *C* respectively, the following relationships hold:

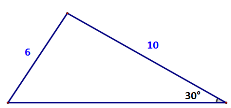
**Law of Cosines**

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$




Notice that the angle is the ***included angle*** between the two known sides. If the ***non-included angle*** is given, the situation is ambiguous as the resulting equation will be quadratic, with two potential solutions.

Consider:



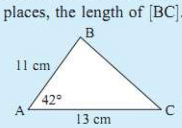

$6^2 = 10^2 + c^2 - 2(10)c\cos 30^\circ$
$36 = 100 + c^2 - 20c(\frac{\sqrt{3}}{2})$
$0 = c^2 - 10\sqrt{3}c + 64$
So $c = 11.98$ or $5.34$

The good news is that using the cosine rule will make it clear that there are multiple solutions because the equation will tell you that!

Some rearranging gives some other useful forms:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

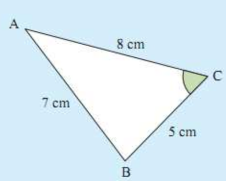
Find, correct to 2 decimal places, the length of [BC].




By the cosine rule:

$\mathrm{BC}^2 = 11^2 + 13^2 - 2 \times 11 \times 13 \times \cos 42^o$
$\therefore \quad \mathrm{BC} \approx \sqrt{(11^2 + 13^2 - 2 \times 11 \times 13 \times \cos 42^o)}$
$\therefore \quad \mathrm{BC} \approx 8.801$
$\therefore$ [BC] is 8.80 cm in length.

In triangle ABC, if AB = 7 cm, BC = 5 cm and CA = 8 cm, find the measure of angle BCA.




By the cosine rule:

$\cos C = \frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}$
$\therefore \quad C = \cos^{-1}\left(\frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}\right)$
$\therefore \quad C = 60^o$

So, angle BCA measures $60^o$.

# C THE SINE RULE

We have shown that the area of a triangle can be given as:

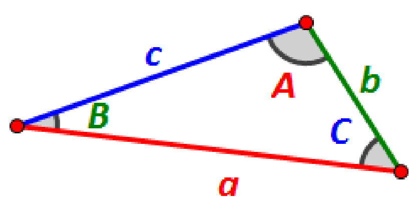
$$\tfrac{1}{2}ab\bullet\sin C = \tfrac{1}{2}ac\bullet\sin B = \tfrac{1}{2}ab\bullet\sin C$$

By multiplying all three expressions by 2 and dividing them all by the product *abc* we get the ***sine rule.***

**The Law of Sines (Sine Rule)**

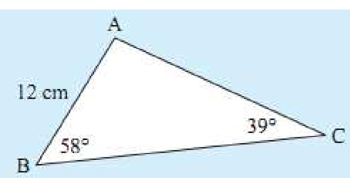
For a triangle with sides of length *a*, *b,* & *c* whose opposite angles are given by *A*, *B*, & *C* respectively, the following relationships hold:

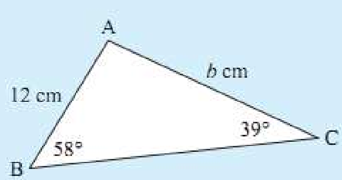
$$\textbf{Law of Sines}\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$




You can use the sine rule to find sides or angles. Some cases are straightforward.

Find the length of [AC] correct to two decimal places.




Using the sine rule, $\dfrac{b}{\sin 58^o} = \dfrac{12}{\sin 39^o}$

$\therefore \quad b = \dfrac{12 \times \sin 58^o}{\sin 39^o}$

$\therefore \quad b \approx 16.170\,74$

$\therefore$ [AC] is about 16.17 cm long.

9B: #1-7 (Cosine Rule)
9C.1: #1-2 (Sine Rule - Finding sides)

Law of Sines (continued) - Choose a partner. Do the investigation on p. 225.
Teams turn in a one page summary of what you learned at the end of the period.

The ***Ambiguous Case*** of the Law of Sines

When given two sides and a ***non-included*** angle, it is possible two have one, two or no solutions to the triangle!
You must explore the situation carefully. Using the cosine rule will help!

## D USING THE SINE AND COSINE RULES

In problems involving ***solving*** triangles, you have three options:
- Try to find right triangles and use definitions
- Use the cosine rule
- Use the sine rule

Often more than one approach can work. Choose the one that is easiest.

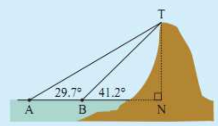
Use the **cosine rule** when given:
- three sides
- two sides and an included angle.

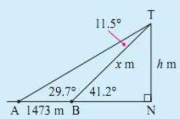
Use the **sine rule** when given:
- one side and two angles
- two sides and a non-included angle, but beware of the *ambiguous case* which can occur when the smaller of the two given sides is opposite the given angle.

The angles of elevation to the top of a mountain are measured from two beacons A and B at sea.

These angles are as shown on the diagram.

If the beacons are 1473 m apart, how high is the mountain?




$A\hat{T}B = 41.2° - 29.7°$ {exterior angle of $\Delta$}
$= 11.5°$

We find $x$ in $\Delta$ABT using the sine rule:

$$\frac{x}{\sin 29.7°} = \frac{1473}{\sin 11.5°}$$

$$\therefore\quad x = \frac{1473}{\sin 11.5°} \times \sin 29.7°$$

$$\approx 3660.62$$

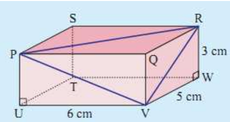
Now, in $\Delta$BNT, $\sin 41.2° = \dfrac{h}{x} \approx \dfrac{h}{3660.62}$

$\therefore\quad h \approx \sin 41.2° \times 3660.62$
$\therefore\quad h \approx 2410$

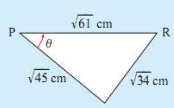
So, the mountain is about 2410 m high.

**Example 9** **Self Tutor**

Find the measure of angle RPV.




In $\Delta$RVW, $\text{RV} = \sqrt{5^2+3^2} = \sqrt{34}$ cm. {Pythagoras}
In $\Delta$PUV, $\text{PV} = \sqrt{6^2+3^2} = \sqrt{45}$ cm. {Pythagoras}
In $\Delta$PQR, $\text{PR} = \sqrt{6^2+5^2} = \sqrt{61}$ cm. {Pythagoras}




By rearrangement of the cosine rule,

$$\cos\theta = \frac{(\sqrt{61})^2 + (\sqrt{45})^2 - (\sqrt{34})^2}{2\sqrt{61}\sqrt{45}}$$

$$= \frac{61+45-34}{2\sqrt{61}\sqrt{45}}$$

$$= \frac{72}{2\sqrt{61}\sqrt{45}}$$

$$\therefore\quad \theta = \cos^{-1}\left(\frac{36}{\sqrt{61}\sqrt{45}}\right) \approx 46.6°$$

$\therefore$ angle RPV measures about 46.6°.

9C.2: #1-8 (Sine Rule - Finding angles)
9D: #2-16 even (Using sin & cos rules)