Chapter 5

Graphing and transforming functions

A FAMILIES OF FUNCTIONS

Much of mathematics is based on just a few functions. There are general principals that apply to ***all*** functions. By understanding them, you can interpret, graph, and analyze a huge number of situations.

Some of the primary ***function families*** are given below. The ***parent function*** or (***mother*** or ***father***) is the simplest form, generally centered at the origin with unit coefficients. The ***family*** consists of variations of the parent function, obtained by changing various coefficients.

Name	*General form*	*Parent*
Linear	$f(x) = ax + b, \quad a \neq 0$	$y = x$
Quadratic	$f(x) = ax^2 + bx + c, \quad a \neq 0$	$f(x) = x^2$
Square Root	$f(x) = \sqrt{x} \quad x \geq 0$	$f(x) = \sqrt{x} \quad x \geq 0$
Cubic	$f(x) = ax^3 + bx^2 + cx + d, a \neq 0$	$f(x) = x^3$
Cube Root	$f(x) = \sqrt[3]{x}$	$f(x) = \sqrt[3]{x}$
Exponential	$f(x) = a^x, \quad a > 0, \quad a \neq 1$	$f(x) = b^x \quad b > 0$
Logarithmic	$f(x) = \log_e x \quad$ or $\quad f(x) = \ln x$	$f(x) = \log_b x \quad x > 0$
Reciprocal	$f(x) = \frac{k}{x}, \quad x \neq 0$	$f(x) = \frac{1}{x} \quad x \neq 0$

It is important to know and recognize the major features of each function family. The general shape of each family should be familiar.

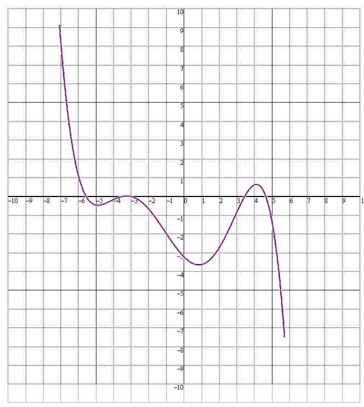
Family	***Parent***	***Domain****	***Range****	***Asymp?***	***Turn Pts. ?***	***X-Int****	***Y-Int****
Linear	$y = x$	$x \in \mathbb{R}$	$y \in \mathbb{R}$	*No*	*No*	1	1
Quadratic	$f(x) = x^2$	$x \in \mathbb{R}$	$y \geq 0$	*No*	*Yes*	0, 1, 2	1
Square Root	$f(x) = \sqrt{x}$	$x \geq 0$	$y \geq 0$	*No*	*No*	1	1*
Cubic	$f(x) = x^3$	$x \in \mathbb{R}$	$y \in \mathbb{R}$	*No*	*Yes*	1, 2, 3	1
Cube Root	$f(x) = \sqrt[3]{x}$	$x \in \mathbb{R}$	$y \in \mathbb{R}$	*No*	*No*	1	1*
Exponential	$f(x) = b^x \quad b > 0$	$x \in \mathbb{R}$	$y > 0$	*Yes*	*No*	0*	1
Logarithmic	$f(x) = \log_b x$	$x > 0$	$y \in \mathbb{R}$	*Yes*	*No*	1	0*
Reciprocal	$f(x) = \frac{1}{x}$	$x \neq 0$	$y \neq 0$	*Yes*	*No*	0*	0*

*The domain, range and number of intercepts is for the ***parent function***. A member of the family may have different values, depending on how it is shifted or stretched.

B TRANSFORMATION OF GRAPHS

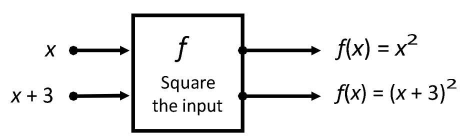
Starting from any function, *f*(*x*), one can ***transform*** it in several ways. Graphically, the most common ***transformations*** are:

Translations - shift the graph horizontally or vertically by some amount
Stretches or compressions - scale the graph vertically or horizontally by some factor
Reflections - flip the graph over a given line
Rotations - rotate the graph around a given point by some angle.



Let's explore the changes in the graph and the changes in the related function.

Before you explore on your own, it's worth reviewing the idea of function composition. Suppose we have a function, *f* that squares whatever comes into it. We would write that as $f(x) = x^2$ since *x* represents "the thing we put into *f*". To help talk about this, we use the word ***argument*** to represent the "expression that we put into a function". So in this case *x* is the argument of *f*. If we put *t* + 5 into *f*, the argument of *f* is *t* + 5 which is written *f*(*t* + 5).



You will practice function composition more in the HW for section 5A.

For the remainder of this period, you will independently explore the relationship between the graph of a function, *f*(*x*) and a ***transformation*** of *f* which we will call *g* that is given by *f*(*x* - *a*) + *b*

The exercises in section 5B.1 should help you understand this. I urge you to use your calculator, graphing both *f* and *g* on the same set of axes to see what happens. Also check out the link on my website to explore these ideas interactively on the web.

AleiMath website

HW: 5A 2,4,5,7,9,10
5B.1 1-4all,6a,7 (Use calculator!)

Summary of ***Translations***

- For $y = f(x) + b$, the effect of b is to **translate** the graph **vertically** through b units.
  - If $b > 0$ it moves **upwards**.
  - If $b < 0$ it moves **downwards**.
- For $y = f(x - a)$, the effect of a is to **translate** the graph **horizontally** through a units.
  - If $a > 0$ it moves to the **right.**
  - If $a < 0$ it moves to the **left**.
- For $y = f(x - a) + b$, the graph is translated horizontally a units and vertically b units. We say it is **translated by the vector** $\binom{a}{b}$.

Translations are created by transforming a function *f*(*x*) into *f*(*x* - *a*) + *b*

Summary of *Translations*

- For $y = f(x) + b$, the effect of b is to **translate** the graph **vertically** through b units.
  - If $b > 0$ it moves **upwards**.
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- For $y = f(x - a)$, the effect of a is to **translate** the graph **horizontally** through a units.
  - If $a > 0$ it moves to the **right.**
  - If $a < 0$ it moves to the **left.**
- For $y = f(x - a) + b$, the graph is translated horizontally a units and vertically b units. We say it is **translated by the vector** $\binom{a}{b}$.

Next, let's explore the effect of the transformations $f(x) = p \cdot f(x)$ and $f(x) = f\left(\frac{x}{p}\right)$

Notice that ***dividing*** *x* by *p* is the same as multiplying *x* by $\frac{1}{p}$

By doing the exercises in 5B.2 and paying attention to the patterns that you see, you should uncover the following properties of ***dilations*** (stretches and compressions).

Summary of *Dilations*

- For $y = pf(x)$, $p > 0$, the effect of p is to **vertically stretch** the graph by the factor p. p is called the **dilation factor**.
  - If $p > 1$ it moves points of $y = f(x)$ **further away** from the x-axis.
  - If $0 < p < 1$ it moves points of $y = f(x)$ **closer** to the x-axis.
- For $y = f\left(\frac{x}{q}\right)$, $q > 0$, the effect of q is to **horizontally stretch** the graph by a factor of q. q is called the **dilation factor**.
  - If $q > 1$ it moves points of $y = f(x)$ **further away** from the y-axis.
  - If $0 < q < 1$ it moves points of $y = f(x)$ **closer** to the y-axis.

The last transformations we'll explore have to do with *f*(-*x*), - *f*(*x*) (*and for the ambitious, -f(-x)*).
Can you guess what the effect of these changes to the function will be? Go through the exercises in 5B.3 to explore.

Summary of *Reflections*

- For $y = -f(x)$, we **reflect** $y = f(x)$ **in the x-axis**.
- For $y = f(-x)$, we **reflect** $y = f(x)$ **in the y-axis**.

HW: 5B.2 1cde,2ab,3,4ac,5ac,7,8 no calc
5B.3 1ace,2,3,4,5,7

There is no new information for section B.4. Just some more practice looking at a variety of transformations and combinations of transformations.

Fri HW: 5B.4 1c,2c,3,4all
QB Practice #1-6