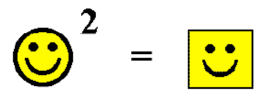
September 06, 2013

Welcome to Pre DP Math

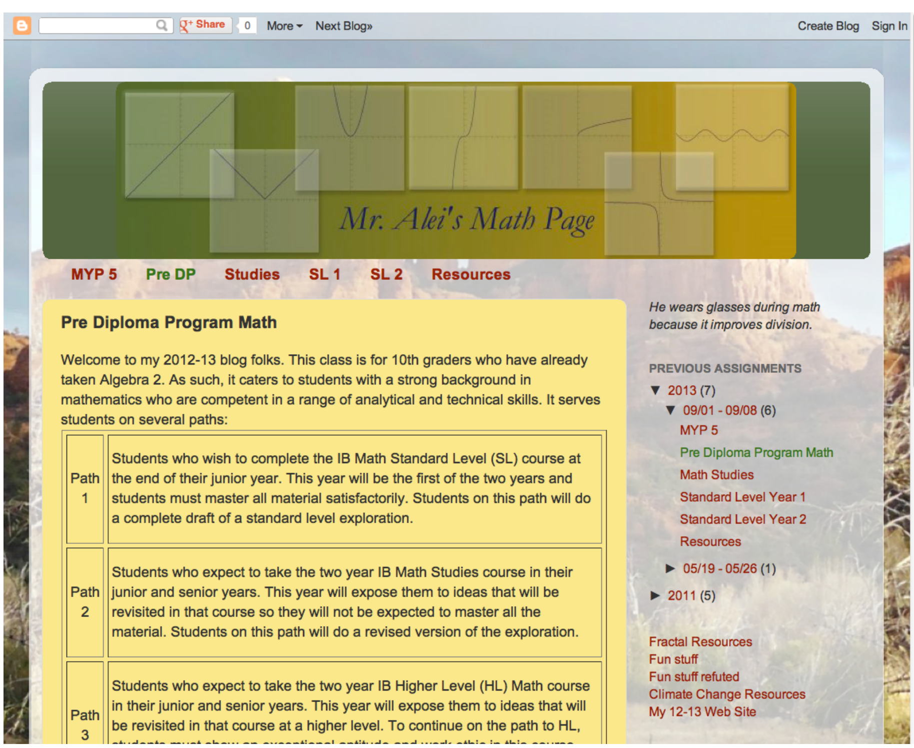


Some things to know:

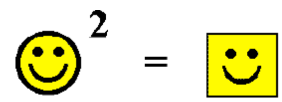
1. Lots of info at www.aleimath.blogspot.com
2. HW - yup. You know you love it! Be prepared to present.
3. Three main paths
   > Take SL 2 next year, IB exams as a junior
   > Take Math Studies 1 next year, IB exams as a senior
   > Take HL 1 next year, IB exams as a senior
4. Content (independent of which path you choose):

	Topic	Hrs	Notes
Topic 1	Algebra	9	Covered in Year 1
Topic 2	Functions and equations	24	Covered in Year 1
Topic 3	Circular functions and trigonometry	16	Covered in Year 1
Topic 4	Vectors	16	To be studied Year 2
Topic 5	Statistics and probability	35	Covered in Year 1
Topic 6	Calculus	40	To be studied Year 2
	Exploration	10	Draft completed Year 1
	Total	150	

5. Grading - Will be adapted to your goals. Quizzes, tests & presentations (80%), Exploration (20%)
6. Bring: Notebooks ($3!), pencil(s), ***calculator*****, and you**!
7. Let's look at the plan in more detail...(Course Overview)
8. Web page tour:
9. Pass out books
10. Key to success: Engage in class, follow through outside of class!



Welcome to IB Math - Standard Level Year 1



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	Total	150	

4. Grading - Ultimately, you need to pass the IB exam! Presentations, quizzes, tests (80%), Exploration (20%)
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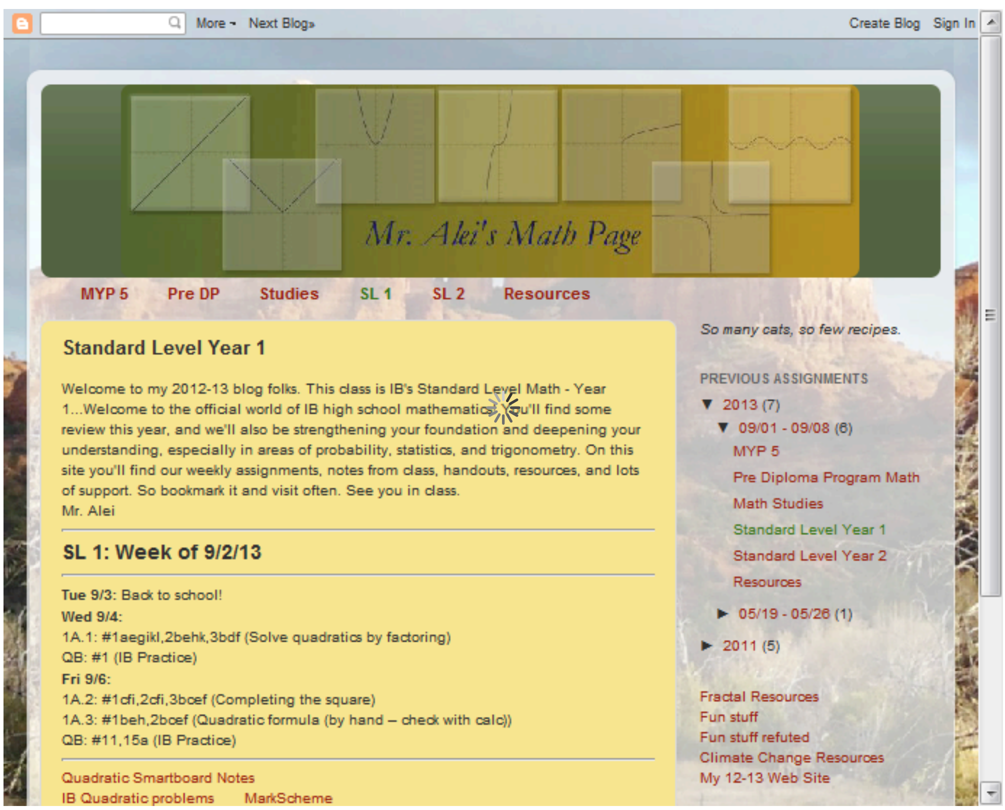


Syllabus Content - Topic 1: Algebra

	Content	Further guidance
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions. Examples include compound interest and population growth.
1.2	Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	*Examples*: $16^{\frac{3}{4}}=8$; $\frac{3}{4}=\log_{16}8$; $\log 32=5\log 2$; $(2^3)^{-4}=2^{-12}$. *Examples*: $\log_4 7=\frac{\ln 7}{\ln 4}$, $\log_{25}125=\frac{\log_5 125}{\log_5 25}\left(=\frac{3}{2}\right)$. Link to 2.6, logarithmic functions.
1.3	The binomial theorem: expansion of $(a+b)^n$, $n\in\mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. **Not required:** formal treatment of permutations and formula for ${}^nP_r$.	Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using **both** the formula and technology. *Example*: finding $\binom{6}{r}$ from inputting $y=6^nC_rX$ and then reading coefficients from the table. Link to 5.8, binomial distribution.

Syllabus Content - Topic 2: Functions

	Content	Further guidance
2.1	Concept of function $f:x\mapsto f(x)$. Domain, range; image (value). Composite functions. Identity function. Inverse function f^{-1}. **Not required:** domain restriction.	*Example*: for $x\mapsto\sqrt{2-x}$, domain is $x\le 2$, range is $y\ge 0$. A graph is helpful in visualizing the range. $(f\circ g)(x)=f(g(x))$. $(f\circ f^{-1})(x)=(f^{-1}\circ f)(x)=x$. On examination papers, students will only be asked to find the inverse of a *one-to-one* function.
2.2	The graph of a function; its equation $y=f(x)$. Function graphing skills. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range. Use of technology to graph a variety of functions, including ones not specifically mentioned. The graph of $y=f^{-1}(x)$ as the reflection in the line $y=x$ of the graph of $y=f(x)$.	**Note** the difference in the command terms "draw" and "sketch". An analytic approach is also expected for simple functions, including all those listed under topic 2. Link to 6.3, local maximum and minimum points.
2.3	Transformations of graphs. Translations: $y=f(x)+b$; $y=f(x-a)$. Reflections (in both axes): $y=-f(x)$; $y=f(-x)$. Vertical stretch with scale factor p: $y=pf(x)$. Stretch in the x-direction with scale factor $\frac{1}{q}$: $y=f(qx)$. Composite transformations.	Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix}3\\-2\end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down. *Example*: $y=x^2$ used to obtain $y=3x^2+2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix}0\\2\end{pmatrix}$.
2.4	The quadratic function $x\mapsto ax^2+bx+c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $x\mapsto a(x-p)(x-q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $x\mapsto a(x-h)^2+k$, vertex (h, k).	Candidates are expected to be able to change from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.
2.5	The reciprocal function $x\mapsto\frac{1}{x}$, $x\ne 0$: its graph and self-inverse nature. The rational function $x\mapsto\frac{ax+b}{cx+d}$ and its graph. Vertical and horizontal asymptotes.	*Examples*: $h(x)=\frac{4}{3x-2}$, $x\ne\frac{2}{3}$; $y=\frac{x+7}{2x-5}$, $x\ne\frac{5}{2}$. Diagrams should include all asymptotes and intercepts.
2.6	Exponential functions and their graphs: $x\mapsto a^x$, $a>0$, $x\mapsto e^x$. Logarithmic functions and their graphs: $x\mapsto\log_a x$, $x>0$, $x\mapsto\ln x$, $x>0$. Relationships between these functions: $a^x=e^{x\ln a}$; $\log_a a^x=x$; $a^{\log_a x}=x$, $x>0$.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.
2.7	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2+bx+c=0$, $a\ne 0$. The quadratic formula. The discriminant $\Delta=b^2-4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solving exponential equations.	Solutions may be referred to as roots of equations or zeros of functions. Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions. *Examples*: $e^x=\sin x$, $x^4+5x-6=0$. *Example*: Find k given that the equation $3kx^2+2x+k=0$ has two equal real roots. *Examples*: $2^{x-1}=10$, $\left(\frac{1}{3}\right)^x=9^{x+1}$. Link to 1.2, exponents and logarithms.
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.

Chapter 1

Quadratics

A QUADRATIC EQUATIONS

Quadratic equations are equations where the highest power of the variable (usually *x*) is two!

Applications:
> Falling bodies
> Accelleration
> Optimization

Quadratic equations come in different flavors (forms)

Standard form:	$ax^2 + bx + c = 0$	$-16t^2 - 16t + 96 = 0$
Factored form:	$a(x - p)(x - q) = 0$	$4(x - 12)(x + 5) = 0$
Vertex Form:	$a(x - h)^2 + k = 0$	$4(x - 12)^2 + 3 = 0$
Disguised Form:	$3e^{2x} - 4e^x + 7 = 0$	$3 + \frac{5}{x} = 2x$ $\frac{2x^2(w+2)}{x} = 5x - \frac{3}{x}$

Solving quadratic equations (like any equation) means...finding the value(s) of the variable that make it true! These values are called ***solutions*** (you will also see the terms ***roots*** or ***zeros***.)

Doing that for quadratics is not as easy as for linear equations - we cannot just manipulate the sides!

There are several techniques that are all helpful in different situations:
> Factoring
> Graphing
> Completing the square
> Using a formula

We will look at these in 1A, beginning with factoring.

Look at a simple equation in factored form: $(x - 1)(x - 2) = 0$ (Remember what a ***factor*** is?!)

We know that if two ***factors*** multiply to zero, one of them must equal zero. This is called the ***zero product property***.

So if $(x - 1)(x - 2) = 0$, then either $(x - 1) = 0$ or $(x - 2) = 0$. These are two ***linear*** equations which we can easily solve using our linear manipulations. Clearly, $x = 1$ or $x = 2$ are the two values we need. So notice that:

A quadratic equation can have two solutions!

If quadratic functions were always given in factored form, there would not be much to discuss. But consider a function in standard form.

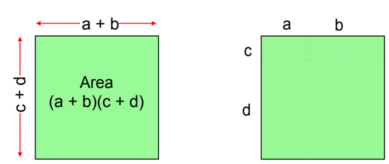
$$x^2 - 6x + 5 = 0$$

This is not so friendly. We can try to ***factor*** the left hand side to find the solutions. In this case, that's not too hard to do. We have to remember ***FOIL*** and do it in ***reverse***.

$$(x - 5)(x - 1) = 0$$

So, *x* is either 5 or 1.

It's really important to understand FOIL well:



F O I L
$(a + b)(c + d) = ac + ad + bc + bd$

F O I L
$(x + b)(x + d) = x^2 + (d + b)x + bd$

Try factoring a few expressions:

$a^2 - 13a + 22$ $q^2 - 11q + 28$ $x^2 - 7x - 18$ $m^2 + 8m - 65$

Below are a bunch of lessons developing and reviewing factoring quadratics. However, rather than take time going through them, I suspect that students have a basic foundation and only really need work with the more difficult situations. So we will review the more complex situations only.

Wouldn't it be lovely if all equations were that easy to factor? Well, yes, but they wouldn't be very useful.
So look at some other situations: (*a*, *b*, and *c* represent the coefficients from standard form)

When *b or c* = 0

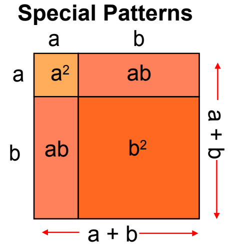
$12x^2 - 15x = 0$

one of the solutions will be zero!

$a^2 - 49 = 0$ $\quad m^2 = 7m$ $\quad u^2 = -9u$ $\quad n^2 - 6n = 0$

When *a* is not equal to one: Several options - takes some experience
> Look for a special pattern
> Check to see if it's factorable
> Directed guess and check (or use the British Method)

Special Patterns



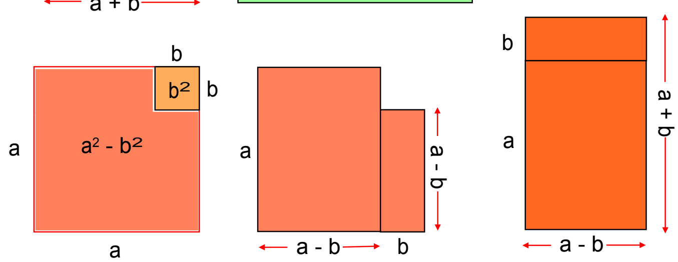
A = (a + b)²

A = a² + 2ab + b²

(a + b)² = a² + 2ab + b²

(a - b)² = a² - 2ab + b²

F L F O I L
(a + b)(a + b) = a² + (ab + ab) + b²
O
= a² + 2ab + b²



a² - b² = (a + b)(a - b)

F L F O I L
(a + b)(a - b) = a² + ab - ab + b²
O
= a² - b²

Pattern	Example
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 4 = (x + 2)(x - 2)$
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 4x + 4 = (x - 2)^2$

You can always do these with reverse FOIL (and you should check). But you will save a ***ton*** of time if you learn these.

$b^2 - 81$ $\quad x^2 - 24x + 144$ $\quad c^2 + 28c + 196$

Special patterns are especially useful when the ***leading coefficient*** *(a)* is not one.

$9p^2 - 12p + 4$ $\quad 64w^2 + 144w + 81$ $\quad 49n^2 - 16$

Remember that ***factoring is a step*** in solving quadratic equations. Sometimes you have to manipulate first.

$4x^2 + 1 = 4x$

1.	$x^2 + 8x + 16$
2. $(x+9)^2$	
3.	$x^2 + x + \frac{1}{4}$
4. $(2x+1)^2$	
5. $(3x+3)^2$	
6.	$\frac{9}{16}x^2 + \frac{1}{2}x + \frac{1}{9}$
7.	$x^2 - 10x + 25$
8.	$x^2 - 14x + 49$
9. $(x - \frac{1}{3})^2$	
10.	$16x^2 - 16x + 4$
11. $(2x-6)^2$	
12.	$\frac{1}{9}x^2 - \frac{1}{2}x + \frac{9}{16}$
13.	$x^2 - 4$
14.	$x^2 - 16$
15.	$x^2 - \frac{1}{4}$
16. $(2x+3)(2x-3)$	
17.	$36x^2 - 25$
18.	$\frac{9}{16}x^2 - \frac{1}{9}$
19.	$16x^2 - 81$
20.	$x^2 - 22x + 121$
21. $(2x+12)^2$	
22.	$16x^2 - 120x + 225$
23. $(8x+3)^2$	
24.	$\frac{9}{16}x^2 - \frac{1}{9}$

Sometimes, you'll have a "hidden" special pattern

***Always Factor out the Greatest Common Factor (GCF) first*!**

Example: Factor $3x^2 + 12x - 15$
Notice that there is a common factor in all three terms!
When there is you should always factor it out first

$3x^2 + 12x - 15$
=
=

Or consider

$5x^3 - 15x^2 - 90x$
=
=

What if GCF and Special Patterns don't work?

Example: Solve $3x^2 + 10x - 8 = 0$

Can I *factor* $3x^2 + 10x - 8$? Well, hmmm...

One way is to guess and check! (it can be tedious, but not always)

Can you reverse think FOIL? (___x + ___)(___x + ___)

The FIRSTS have to multiply to give 3. So they must be

The LASTS have to have different signs and multiply to give 8. Possibilities?

The OUTERS plus INNERS create the x term.

So the only possibilities are:
$(3x - 1)(x + 8)$ or $(3x + 8)(x - 1)$
$(3x + 1)(x - 8)$ or $(3x - 8)(x + 1)$
$(3x - 2)(x + 4)$ or $(3x + 4)(x - 2)$
$(3x + 2)(x - 4)$ or $(3x - 4)(x + 2)$

Which one will add to a middle term of $+10x$? That's your factorization.

In this case, it's so the solutions to the equation are 2/3 and -4.

That's all well and good. But what about:
$15x^2 - 2x - 8 = 0$

The firsts have to multiply to give 15. So they must be

The lasts have to have different signs and multiply to give 8

So the "only" possibilities are:

$(15x - 1)(x + 8)$ or $(15x + 8)(x - 1)$
$(15x + 1)(x - 8)$ or $(15x - 8)(x + 1)$
$(15x - 2)(x + 4)$ or $(15x + 4)(x - 2)$
$(15x + 2)(x - 4)$ or $(15x - 4)(x + 2)$

$(5x - 1)(3x + 8)$ or $(5x + 8)(3x - 1)$
$(5x + 1)(3x - 8)$ or $(5x - 8)(3x + 1)$
$(5x - 2)(3x + 4)$ or $(5x + 4)(3x - 2)$
$(5x + 2)(3x - 4)$ or $(5x - 4)(3x + 2)$

Isn't this going a bit far?

When a and c are not prime numbers, there is a better way!

Reconsider $15x^2 - 2x - 8 = 0$

1) Calculate ac $ac = -120$
2) Find a magic pair that multiplies to ***ac*** and adds to b -12 & 10
3) Rewrite the original expression using a sum for b
$= 15x^2 - 12x + 10x - 8$
4) Factor by grouping: (do the "*Groupie Groupie*")
Factor out the ***GCF*** from the first two terms $= 3x(5x - 4) + 10x - 8$
Factor the "twin" from the last two terms! $= 3x(5x - 4) + 2(5x - 4)$
Gather the *GOOP* (**G**arbage **O**utside **O**f **P**arentheses) $= (3x + 2)(5x - 4)$
5) Check using FOIL
$(3x + 2)(5x - 4) = 15x^2 + 10x - 12x - 8 = 15x^2 - 2x - 8$ Voila!

This is also known as the *British Method***.**

Let's try that again....
Example 2: Solve $12x^2 + 5x - 7 = 0$

1) Calculate ac $ac = -84$

2) Find a magic pair that multiplies to ***ac*** and adds to b 12 & -7

3) Rewrite the original expression using a sum for b
$= 12x^2 + 12x - 7x - 7$

4) Factor by grouping: (do the "*Groupie Groupie*")
Factor out the ***GCF*** from the first two terms $= 12x(x + 1) - 7x - 7$

Factor the "twin" from the last two terms! $= 12x(x + 1) - 7(x + 1)$

Gather the *GOOP* (**G**arbage **O**utside **O**f **P**arentheses) $= (12x - 7)(x + 1)$

5) Check using FOIL
$(12x - 7)(x + 1) = 12x^2 - 7x + 12x - 7 = 12x^2 + 5x - 7$ Yay!

6) Find the ***solutions to the equation***:
$(12x - 7)(x + 1) = 0$ when $x = -1$ or when $12x - 7 = 0$......$x = 7/12$

Try one:

Solve $8x^2 – 6x – 9 = 0$

Summary of Factoring Process

1) Check to see if you can factor (discriminant)
2) Factor out a GCF
3) Look for special patterns
4) Use reverse FOIL if a = 1
5) Use Guess & Check or British Method if a ≠ 1

Factoring $ax^2 + bx + c$ when $a \neq 1$

Step	Example	Example	Example	Example	Example
Step	$6x^2 + 17x + 12$	$-10x^2 + 14x - 4$	$6x^2 + 18x + 12$	$4x^2 + 20x + 25$	$28x^2 - 63$
1. Find the discriminant first. If it's not a perfect square, stop! You can't factor into integers!	$b^2 - 4ac = 289 - 288$ $= 1$ … Keep going!	$b^2 - 4ac = 196 - 160$ $= 36$ … Keep going!	$b^2 - 4ac = 324 - 288$ $= 36$ Keep going!	$b^2 - 4ac = 400 - 400$ $= 0$ … Keep going!	$b^2 - 4ac = 0 + 7056$ $= 7056 = (84^2)$… Keep going!
2. Factor out a −1 and the Greatest Common Factor from all terms.	There is no GCF	$= -2\,(5x^2 - 7x + 2)$ *(Honey, I shrunk the kids!)*	$= 6\,(x^2 + 3x + 2)$	There is no GCF	$= 7(4x^2 - 9)$
3. Is this a special pattern?	No	No	No	Yes!	Yes!
4. Compute **ac** and **b**	**ac** = 72, **b** = 17	**ac** = 10, **b** = −7	Hey, **a** = 1!	$= (2x + 5)^2$	$= 7(2x + 3)(2x - 3)$
5. Find a magic pair that multiplies to give **ac** and adds to give **b**. If no pair exists, the expression cannot be factored using integers. Be sure to check them **all** before reaching this conclusion.	**ac = 72** | **b = 17** 1, 72 2, 36 3, 24 4, 18 6, 12 8, 9 | *	**ac = 10** | **b = −7** 1, 10 2, 5 −2, −5 | * List *all* the factors of **ac** and find two that add to give **b**.	So **ac** is just **c** and we use the simple method. What multiplies to give **c** and adds to give **b**? *(Answer: 2 and 1)* $= 6\,(x + 2)(x + 1)$	**Finished!** (aren't you glad you practiced those special patterns!)	**Finished!** (aren't you glad you practiced those special patterns!)
6. Rewrite the original expression with four terms using the magic pair instead of **b**.	$= 6x^2 + 9x + 8x + 12$ Don't worry about the order of the two x terms. It doesn't matter.	$= -2\,[5x^2 - 2x - 5x + 2]$ If you multiply this out at this step, you should get the original expression. Do you?	**Finished!**	*Mathematically speaking, steps 6 & 7 combined are called "factor by grouping".*	
7. Factor out the GCF from the first two terms.	$= \mathbf{3x(2x + 3)} + 8x + 12$ *(Let me introduce my twin!)*	$= -2\,[\mathbf{x(5x - 2)} - 5x + 2]$	*Do the Groupie Groupie!*		
8. Factor out the GCF from the last two terms. Factor out a −1 if the third term is negative.	$= 3x(2x + 3) + \mathbf{4(2x + 3)}$ $= \boxed{3x}\,(2x + 3) + \boxed{4}\,(2x + 3)$	$= -2\,[x(5x - 2) - \mathbf{1(5x - 2)}]$ Since the 3rd term $(-5x)$ was negative, we factor out a −1.			
9. You end up with the same binomial. Factor out the binomial.	$= (3x + 4)(2x + 3)$	$= -2\,(x - 1)\,\mathbf{(5x - 2)}$ *Don't forget the −2 sitting in the peanut gallery…*	*Gather GOOP – Garbage Outside Of Parentheses and the twin!*		
10. Check your work by multiplying with FOIL.	$= 6x^2 + 17x + 12$	$= -2(5x^2 - 7x + 2)$ $= -10x^2 + 14x - 4$			

When done, re-read the problem. Are you *solving an equation*? If so, there is another step or two. If you're just *factoring*, you're done.

Factoring Practice: a<>1

1. $2x^2-5x-12$	6. $3x^2+10x+8$	11. $12x^2+7x+1$	16. $3x^2-8x-16$
2. $9x^2-15x+4$	7. $3x^2-7x+4$	12. $3x^2+7x+4$	17. $3x^2-11x-4$
3. $2x^2-7x+3$	8. $8x^2-18x+9$	13. $9x^2-15x+4$	18. $2x^2+5x-3$
4. $2x^2-x-6$	9. $12x^2+25x+12$	14. $3x^2+13x+12$	19. $2x^2+3x-2$
5. $3x^2+13x+4$	10. $9x^2-24x+16$	15. $2x^2+9x+4$	20. $6x^2+11x+3$

Solve the equation

32. $16x^2 - 1 = 0$ **33.** $11q^2 - 44 = 0$ **34.** $14s^2 - 21s = 0$

35. $45n^2 + 10n = 0$ **36.** $4x^2 - 20x + 25 = 0$ **37.** $4p^2 + 12p + 9 = 0$

38. $15x^2 + 7x - 2 = 0$ **39.** $6r^2 - 7r - 5 = 0$ **40.** $36z^2 + 96z + 15 = 0$

Factoring to solve quadratic equations

1. Start by rearranging to standard form
2. Remember that $x = 0$ might be a solution!
3. Factor out any GCF
   > Careful with dividing by x since $x = 0$ may be a solution
4. Consider removing fractions by multiplying through by a common denominator
5. Look for special patterns
   > Square of a sum or difference?
   > Difference of two squares?
6. Look at the discriminant - is it worth trying?
7. If $a \neq 1$, consider guess & check or British method

Try these situations:

$$9x^2 - 12x + 4 = 0$$

$$2x^2 - 11x = 0$$

$$4x^2 = 11x + 3$$

$$x^2 = 2x + 8$$

Sometimes the equation doesn't come so nicely packaged. You need to recognize ***when*** an equation is quadratic and be able to rewrite it in a familiar form. Are these equations quadratic?
If so, write them in standard form.

$$x^2 + (3k - 1)x + (2k + 10) = 0$$

$$4x^2 - 3x + x(5 - 4x) = 6 + \frac{2}{x}$$

$$x(x^2 - 5) = 3x + 5x^2 + x^3$$

$$\frac{2x^2(x + 2)}{x} = 5x^2 - \frac{3}{x}$$

$$\frac{x + 2}{x - 9} = 4x$$

$$3 + \frac{5}{x} = 2x$$

We could go on, but you get the point! Accurate algebraic manipulations are ***key***!

1A.1: #1aegikl,2behk,3bdf (Solve quadratics by factoring)
QB: #1 (IB Practice)

So you can factor an equation like $x^2 - 5x - 84 = 0$ into and find solutions

Suppose now, that the equation is $x^2 - 81 = 0$. Would you factor?

How about $x^2 - 12 = 0$. What are the solutions now?

Now look at $m^2 - k = 0$. Solutions?

What about $(x - 3)^2 - 4 = 0$?

Graph the function $f(x) = (x - 3)^2 - 4$. Where do you think the ***zeros*** are?

Or solve $3(t + 5)^2 + 4 = 13$

What is common about all of these situations?

Solving by Square Roots

When the equation is written with no linear term, you can solve by taking a square root.

So let's try something like $x^2 + 6x = 16$. Can we take a square root of both sides?

What would you do so that you ***could*** take the square root of both sides?

By adding 9 to both sides we get a nice result: $x^2 + 6x + 9 = 25$ or $(x + 3)^2 = 25$

Now we can take the square root to get: $x = \pm 5 - 3$ or $x = 2$ or -8

$x^2 + 6x + 9$ is called a ***perfect square trinomial*** because it can be written as $(x \pm a)^2$

(a) $x^2 - 12x + 36$ (b) $x^2 + 14x + 49$ (c) $x^2 - 20x + 100$

As suggested, these should all look like either $(x - r)^2$ or $(x + r)^2$. State the important connection between the *coefficients* of the given trinomials and the values you found for r.

5. (Continuation) In the following, choose k to create a perfect-square trinomial:

(a) $x^2 - 16x + k$ (b) $x^2 + 10x + k$ (c) $x^2 - 5x + k$

What value of k will make the following perfect squares?

$x^2 + bx + k$ $x^2 - bx + k$

Solve these equations by ***completing the square***.

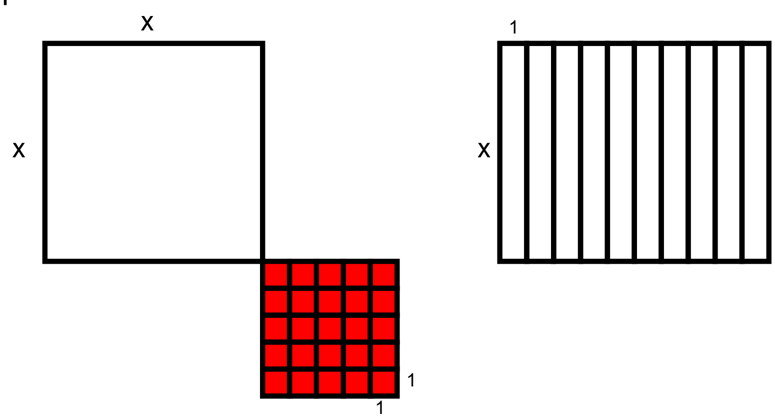
(a) $x^2 - 8x = 3$ (b) $x^2 + 10x = 11$ (c) $x^2 - 5x - 2 = 0$ (d) $x^2 + 1.2x = 0.28$

Completing the square

Rewrite an equation so that there is no linear term!
Get the variable into a "perfect square".

$\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$ may come in handy!

Suppose I have $x^2 + 10x$ and I want to add some number of 1's to "complete a perfect square"



6.A.3 The Quadratic Formula

Let's use completing the square to find a formula for the solution to ***any*** quadratic equation!

You need to be able to recreate this: Take careful notes and follow carefully:

Start with standard form:	$ax^2+bx+c=0$
Divide both sides by a:	$x^2+\frac{b}{a}x+\frac{c}{a}=0$
Subtract $\frac{c}{a}$ from both sides:	$x^2+\frac{b}{a}x=-\frac{c}{a}$
Complete the square on the LHS:	$x^2+\frac{b}{a}x+\left(\frac{1}{2}\frac{b}{a}\right)^2=-\frac{c}{a}+\left(\frac{1}{2}\frac{b}{a}\right)^2$
Clean up the RHS: Now since the LHS is a perfect square:	$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$ $=\frac{b^2}{4a^2}-\frac{c}{a}$ $=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$ $=\frac{b^2-4ac}{4a^2}$ $\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$
Time to take the square root:	$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$
Get x all alone:	$x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}$
Remove the denominator in the radical:	$x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}$ $=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$ $=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$
The Quadratic Formula:	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

For any quadratic equation in standard form $ax^2+bx+c=0$ the solutions, x, are given by

The Quadratic Formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

A quadratic is more generally known as a 2nd degree polynomial, so on a calculator you can use the Polynomial and Simultaneous Equation solver: PLYSMLT2!

1A.2: #1cfi,2cfi,3bcef (Completing the square)
1A.3: #1beh,2bcef (Quadratic formula (by hand – check with calc))
QB: #11,15a (IB Practice)

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B THE DISCRIMINANT OF A QUADRATIC

Recall the quadratic formula. Let's look at it in more detail.

For any quadratic equation in standard form $ax^2 + bx + c = 0$
the solutions, x, are given by

The Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Let's do an example:

$$3x^2 - 4x + 5 = 0$$

Solution: $x = \dfrac{+4 \pm \sqrt{(-4)^2 - 4(3)(5)}}{2(3)} = \dfrac{4 \pm \sqrt{16-60}}{6}$

$$= \frac{4 \pm \sqrt{-44}}{6}$$

$$= \frac{4 \pm 2i\sqrt{11}}{6}$$

$$= \frac{2 \pm i\sqrt{11}}{3}$$

$$or\ \frac{2 + i\sqrt{11}}{3}\ and\ \frac{2 - i\sqrt{11}}{3}$$

Which leads us to looking at the expression inside the radical: Notice that:

If $b^2 - 4ac > 0$, the radical is real. There are two real solutions.
If $b^2 - 4ac < 0$, the radical is imaginary. There are two complex solutions.

If $b^2 - 4ac = 0$, the radical is zero. There is one real solution at $x = \dfrac{-b}{2a}$

If $b^2 - 4ac$ is a perfect square, you can factor the quadratic into integers!
If not, you can't!

$b^2 - 4ac$ is called the ***discriminant*** because it discriminates between types of solutions.
It's nice to know what's going to happen before you get started!

IB uses the symbol Δ to represent the discriminant

Try some:

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

4. $2x^2 + 4x - 4 = 0$ **5.** $3x^2 + 12x + 12 = 0$ **6.** $8x^2 = 9x - 11$

7. $7x^2 - 2x = 5$ **8.** $4x^2 + 3x + 12 = 3 - 3x$ **9.** $3x - 5x^2 + 1 = 6 - 7x$

Another possible situation:

For the equation $kx^2 + (k+3)x = 1$ find the discriminant Δ and draw a sign diagram for it. Hence, find the value of k for which the equation has:

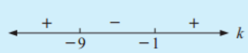
a two distinct real roots **b** two real roots

c a repeated root **d** no real roots.

For $kx^2 + (k+3)x - 1 = 0$, $a = k$, $b = (k+3)$, $c = -1$

So, $\Delta = b^2 - 4ac$
$= (k+3)^2 - 4(k)(-1)$
$= k^2 + 6k + 9 + 4k$
$= k^2 + 10k + 9$
$= (k+9)(k+1)$

and has sign diagram:



a For two distinct real roots, $\Delta > 0$ $\therefore$ $k < -9$ or $k > -1$.

b For two real roots, $\Delta \geqslant 0$ $\therefore$ $k \leqslant -9$ or $k \geqslant -1$.

c For a repeated root, $\Delta = 0$ $\therefore$ $k = -9$ or $k = -1$.

d For no real roots, $\Delta < 0$ $\therefore$ $-9 < k < -1$.

A quadratic equation $9x^2 - 5x + k = 0$ has exactly one solution. Find k.

..."touches the x - axis exactly once."

..."has two identical solutions."

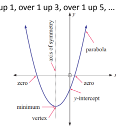
1B: #2all,3all, 4bdf (Discriminant)
QB: #12,14 (IB Practice)

C GRAPHING QUADRATIC FUNCTIONS

When we graph a quadratic ***function*** we get a...***parabola***.

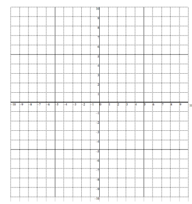
Features of the Mother function $y = x^2$
> Vertex at origin
> Pattern from origin is over 1 up 1, over 1 up 3, over 1 up 5, ... over 1 up by odds

Features of ***all*** parabolas
> Symmetric



Graphing from different forms:
Factored Form $y = a(x - p)(x - q)$
Vertex Form $y = a(x - h)^2 + k$
Standard form $y = ax^2 + bx + c$

Graph $f(x) = 2(x - 4)(x + 2)$



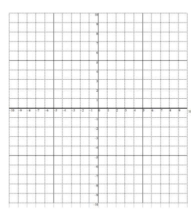
Graphing from *factored form*
> Graph the zeros
> Graph the axis of symmetry (midway between zeros)
> Find the vertex (plug x of the axis of symmetry into the function to find $f(x)$)
> Sketch the curve. y-intercept is at apq

Try these. Sketch showing zeros, vertex, and axis of symmetry.

$y = (x-4)(x+2)$ $y = -3x(x+4)$ $y = 2(x+3)^2$

Graphing from different forms:
Factored Form $y = a(x - p)(x - q)$
Vertex Form $y = a(x - h)^2 + k$
Standard form $y = ax^2 + bx + c$

Graph $f(x) = 2(x - 4)^2 - 3$



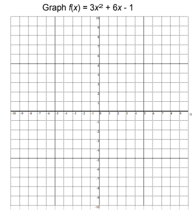
Graphing from *vertex form*
> Graph the vertex and axis of symmetry
> Use a to identify the direction and width (roughly)
> Find and graph the y-intercept ($y = ah^2 + k$) and it's symmetric point
> Find the zeros by taking square roots.
> Sketch the curve.

Try these. Sketch showing vertex, axis of symmetry, y-intercept, and approximate zeros.

$y = (x-1)^2 + 3$ $y = -2(x-1)^2 - 3$ $y = \frac{1}{2}(x-3)^2 + 2$

Graphing from different forms:
Factored Form $y = a(x - p)(x - q)$
Vertex Form $y = a(x - h)^2 + k$
Standard form $y = ax^2 + bx + c$

Graph $f(x) = 3x^2 + 6x - 1$



Graphing from *standard form*

> Try factoring first. If possible, graph from factored form.

> Find and graph the axis of symmetry. $x = \frac{-b}{2a}$

> Find and graph the vertex (evaluate f at the axis of symmetry).
> Find and graph the y-intercept ($y = c$) and it's symmetric point.
> Use quadratic formula to find zeros if you really want accuracy.
> Sketch the curve. Test the approximate zeros in the function.

Try these. Sketch showing vertex, axis of symmetry, y-intercept, and zeros if factorable.

$y = x^2 + 7x + 10$ $y = 2x^2 - 5x + 2$ $y = -2x^2 + x + 1$

<u>Graphing "strategy"</u>

We've discussed graphing from different forms - what strategies help us do it ***efficiently***?

1. Completing the square
2. Using the discriminant

Completing the square to help graph a function

Suppose we want to graph the function $f(x) = x^2 + 6x - 4$

It would be nice to write it in the form $f(x) = a(x - h)^2 + k$. Why?

So notice that	$f(x) = x^2 + 6x - 4$	can be written as:
Add and subtract 9 to the RHS	$f(x) = x^2 + 6x + 9 - 9 - 4$	which now has a "perfect square"
Rewrite with a binomial squared	$f(x) = (x + 3)^2 - 13$	we now have "vertex form"

We can now graph the vertex at (-3, -13). We can find the roots by finding the x values where the function is zero:

Set the function value to zero	$0 = (x + 3)^2 - 13$
Add 13 to both sides	$13 = (x + 3)^2$
Take the square root (don't forget ±)	$\pm\sqrt{13} = x + 3$
Subtract 3 and we're done	$x = -3 \pm \sqrt{13}$

It's also easy to find the y-intercept
- From the original function in standard form or by plugging x=0 into the vertex form of the function

So the y-intercept is -4

Completing the Square

A quadratic form of $x^2 + bx$ can be made into a perfect square binomial by adding $\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$ The resulting perfect square binomial is $\left(x + \frac{b}{2}\right)^2$

Let's add a twist: What happens if the ***leading coefficient*** (a) is not one!

Short answer: Factor out a!

Consider this one	$f(x) = 2x^2 - 16x + 7$	
Factor out the leading coefficient	$f(x) = 2[x^2 - 8x + \frac{7}{2}]$	Don't panic with fractions!
Add and subtract $\left(\frac{b}{2}\right)^2$ to complete the square	$f(x) = 2[x^2 - 8x + 16 - 16 + \frac{7}{2}]$	
Combine the left overs (fractions again)	$f(x) = 2[(x-4)^2 - \frac{32}{2} + \frac{7}{2}] = 2[(x-4)^2 - \frac{25}{2}]$	
Distribute the leading coefficient back through	$f(x) = 2(x-4)^2 - 25$	

From here you can graph as we did above.

Another example: Rewrite $f(x) = 3x^2 - 4x + 1$ in vertex form.

$f(x) = 3x^2 - 4x + 1$

$f(x) = 3[x^2 - \frac{4}{3}x + \frac{1}{3}]$ Honey, I shrunk the kids

$f(x) = 3[x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9} + \frac{1}{3}]$ Don't panic with fractions!

$f(x) = 3[(x - \frac{2}{3})^2 - \frac{1}{9}]$

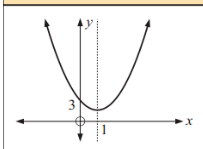
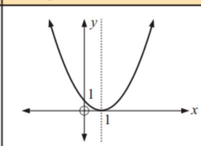
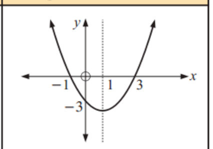
$f(x) = 3(x - \frac{2}{3})^2 - \frac{1}{3}$ Bring the kids back!

1C.1: #1bcf,2,3,4bef,5,6cfi,8beh (Sketching parabolas)
1C.2: #2bdf,3cf (Graph from vertex form (completing the square))

Recall the characteristics of the discriminant:

> If b^2 - 4ac > 0, the radical is real. There are two real solutions.
> If b^2 - 4ac < 0, the radical is imaginary. There are two complex solutions.
> If b^2 - 4ac = 0, the radical is zero. There is one real solution at $x = \dfrac{-b}{2a}$.
>
> If b^2 - 4ac is a perfect square, you can factor the quadratic into integers.
> If not, you can't!

IB uses the symbol Δ to represent the discriminant

$y = x^2 - 2x + 3$	$y = x^2 - 2x + 1$	$y = x^2 - 2x - 3$
		
$\Delta = b^2 - 4ac$ $= (-2)^2 - 4(1)(3)$ $= -8$	$\Delta = b^2 - 4ac$ $= (-2)^2 - 4(1)(1)$ $= 0$	$\Delta = b^2 - 4ac$ $= (-2)^2 - 4(1)(-3)$ $= 16$
$\Delta < 0$	$\Delta = 0$	$\Delta > 0$
does not cut the x-axis	touches the x-axis	cuts the x-axis twice

If there is one solution, then Δ = 0. So we can answer questions like:

The function $f(x) = kx^2 - 8x + 2$ has a repeated root. Find the value of k.

Or a more complex one: $f(x) = kx^2 - (k + 1)x - 3$ touches the x axis once. Find the value(s) of k.

Solution:

Since there is one root, $b^2 - 4ac = 0$

So $[-(k + 1)]^2 - 4(k)(-3) = 0$

or $k^2 + 2k + 1 - 12k = 0$ leading to $k^2 - 10k + 12 = 0$

Use quadratic formula or complete the square to solve $k^2 - 10k + 12 = 0$

$$k = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(12)}}{2(1)}$$

$$k == \frac{10 \pm \sqrt{100 - 48}}{2} = 5 \pm \frac{\sqrt{52}}{2} = 5 \pm \sqrt{13}$$

Positive Definite and Negative Definite

A quadratic is "Positive definite" if all of it's values are positive (not zero!).
Happens when: a > 0 (opens up) and discriminant is negative (no roots)

A quadratic is "Negative definite" if ***all*** of it's values are negative (not zero!).
Happens when: a < 0 (opens down) and discriminant is negative (no roots)

1C.3: #1cf,2bd,3,4 (Discriminant in graphing)
QB: #3,4,5,6,8 (IB Practice)