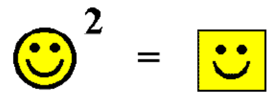
Welcome to IB Math - Standard Level Year 2

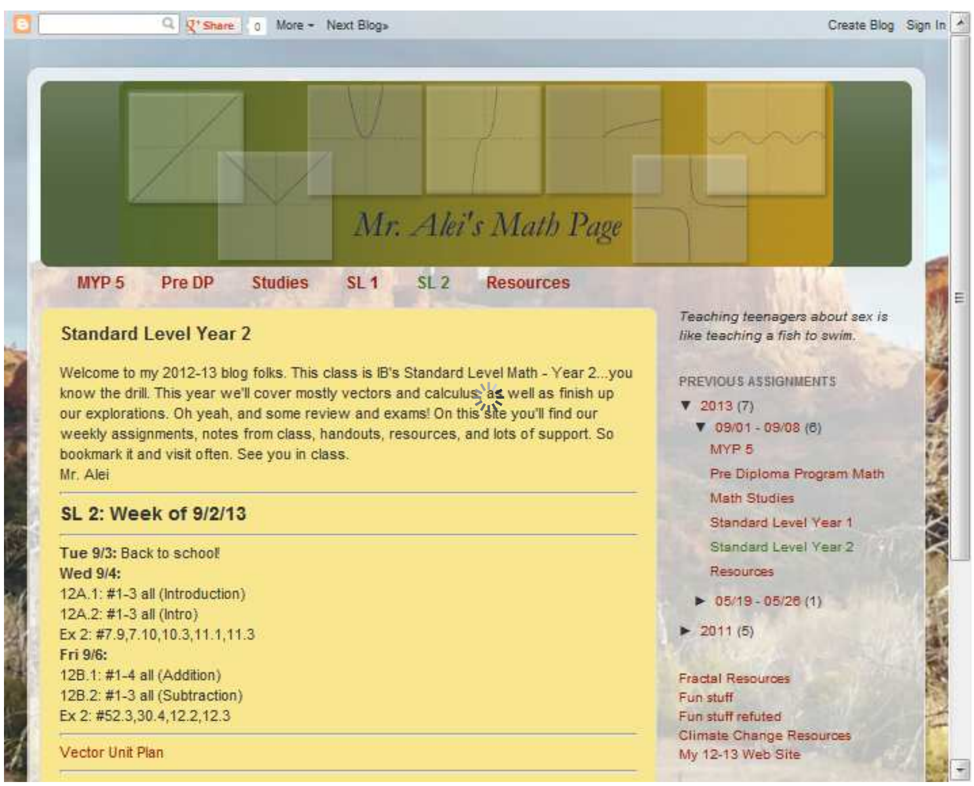
Some things to know:



1. Lots of info at www.aleimath.blogspot.com
2. HW - yup. You know you love it! Be prepared to present.
3. Content:

	Topic	Hrs	Notes
Topic 1	Algebra	9	Covered in Year 1
Topic 2	Functions and equations	24	Covered in Year 1
Topic 3	Circular functions and trigonometry	16	Covered in Year 1
Topic 4	**Vectors**	**16**	**To be studied Year 2**
Topic 5	Statistics and probability	35	Covered in Year 1
Topic 6	**Calculus**	**40**	**To be studied Year 2**
	Exploration	10	Draft completed Year 1 **Final due before Winter Break**
	Total	**150**	

4. Grading - Ultimately, you need to pass the IB exam!
   Presentations, quizzes, tests (80%), Exploration (20%)
5. Bring: Notebooks ($3!), pencil(s), ***calculator,*** **and you**!
6. Let's look at the plan in more detail...
7. Web page tour:
8. Pass out books



	Content	Further guidance
4.1	Vectors as displacements in the plane and in three dimensions.	Link to three-dimensional geometry, x, y and z-axes.
	Components of a vector; column representation; $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ (standard basis).
	Algebraic and geometric approaches to the following:	Applications to simple geometric figures are essential.
	• the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.
	• multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;	Multiplication by a scalar can be illustrated by enlargement.
	• magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;	
	• unit vectors; base vectors; $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$;	
	• position vectors $\vec{\mathrm{OA}} = \boldsymbol{a}$;	
	• $\vec{\mathrm{AB}} = \vec{\mathrm{OB}} - \vec{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$.	Distance between points A and B is the magnitude of $\vec{\mathrm{AB}}$.
4.2	The scalar product of two vectors.	The scalar product is also known as the "dot product". Link to 3.6, cosine rule.
	Perpendicular vectors; parallel vectors.	For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\boldsymbol{w} = k\boldsymbol{v}$, $\lvert\boldsymbol{v} \cdot \boldsymbol{w}\rvert = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.
	The angle between two vectors.	
4.3	Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.	Relevance of $\boldsymbol{a}$ (position) and $\boldsymbol{b}$ (direction). Interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert\boldsymbol{b}\rvert$ representing speed.
	The angle between two lines.	

Chapter 12
Vectors

This summer I went to a math conference in Phoenix. I flew from Santa Fe to Albuquerque. From there, I went on to Phoenix. Draw a diagram that illustrates the path I took. Also show the path I could have taken if I had flown directly from Santa Fe to Phoenix.

Eric drove off the road and got stuck in the mud. He starts pushing from the front of the car with a force of 200 pounds. His date, Emily, stands next to him and lifts straight up on the bumper with a force of 100 pounds. Draw a side view diagram that illustrates the forces on the car. Imagine the "total" force being applied to the care by both Eric and Emily. Can you illustrate that in your diagram?

Do you notice anything mathematically special about the above situations?

They both include ideas that involve both an ***amount*** and a ***direction***. We need a word for these kinds of quantities.

Vectors and ***Scalars***
A ***vector*** is a quantity that has both ***magnitude*** (amount) and ***direction.***
A ***scalar*** is a quantity that has only ***magnitude.***

How do we represent vectors?

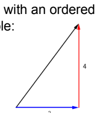
1. One way is with a ***directed line segment***, also known as an ***arrow***.
- The length of the arrow defines the magnitude (remember complex numbers?)
- The direction of the arrow defines the direction of the vector.

Some conventions:
- The length is either labelled or implied by drawing the vector on a grid with known scale
- The direction is often given as an angle measured in a given direction from a given reference
  - > Examples: 20° CCW from horizontal, 15° North of West

2. Another way to represent vectors is with an ordered pair that gives the horizonal and vertical components of the vector. For example:

(3, 4) represents the vector

This is also written [3, 4] or $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$



In books, letters that represent vectors are often bolded and italicized, sometimes with arrows above them. For exampl
$\boldsymbol{v}$ = [3, 4] or $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Students should indicate a vector with an arrow above it: $\vec{v}$ or $\vec{\boldsymbol{v}}$

The starting point of the vector is called the ***tail***, the ending point is the ***head***. Notice that such vectors do not exist at any particular place in space! They simply describe a magnitude and direction of change or ***displacement*** from the tail to the head. They are thus called:

Displacement Vectors
... can be located anywhere in space

3. A third way to represent a vector is as a segment between two points with specified locations. Often the initial point is the ***origin***, known as ***O***. The vector from the origin to point *A* is written $\overrightarrow{OA}$ and is called the ***position vector*** of point *A* since it defines a precise position in space where the vector is located.

If the originating point is ***not*** the origin, we write $\overrightarrow{AB}$ which represents the vector ***from*** *A* ***to*** *B*. (Notice that this is ***not*** the same as the vector $\overrightarrow{BA}$ ...why not?) The vector $\overrightarrow{AB}$ is called the ***position vector of B relative to A***.

Position Vectors
... are located at defined points in space
... are "relative" to their starting point
... are just called "position vectors" when they are relative to the origin

Your turn. ***v*** and ***w*** are vectors. Talk to your neighbor and decide what you think is meant by:
» ***w* = *v***
» ***w* = -*v***
» **|*v*|**

Let's be more specific. Let ***v*** be the vector [3, 4]. Find the vector ***w*** if:
» ***w* = *v***
» ***w* = -*v***

The Opposite of a Vector
... has coordinates that are the opposite of the original
... is parallel to and in the opposite direction of the original

12A.1: #1-3 all (Introduction)
12A.2: #1-3 all (Intro)

B GEOMETRIC OPERATIONS WITH VECTORS

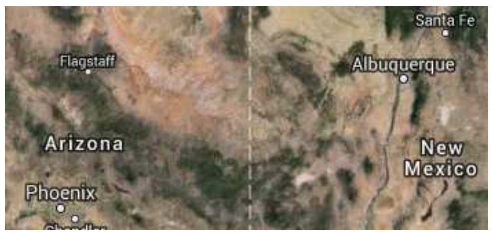
9. Draw the following segments. What do they have in common?
from $(3,-1)$ to $(10,3)$; from $(1.3, 0.8)$ to $(8.3, 4.8)$; from $(\pi, \sqrt{2})$ to $(7+\pi, 4+\sqrt{2})$.

10. (Continuation) The *directed segments* have the *same* length and the *same* direction. Each represents the *vector* $[7, 4]$. The *components* of the vector are the numbers 7 and 4.
(a) Find another example of a directed segment that represents this vector. The initial point of your segment is called the *tail* of the vector, and the final point is called the *head*.
(b) Which of the following directed segments represents $[7, 4]$? from $(-2,-3)$ to $(5,-1)$; from $(-3,-2)$ to $(11,6)$; from $(10,5)$ to $(3,1)$; from $(-7,-4)$ to $(0,0)$.

3. Give the components of the vector whose length is 10 and whose direction *opposes* the direction of $[-4, 3]$.

1. Instead of saying that Cary moves *3 units left and 2 units up*, you can say that Cary's position is *displaced* by the vector $[-3, 2]$. Identify the following displacement vectors:
(a) Stacey starts at $(2,3)$ at 1 pm, and has moved to $(5,9)$ by 6 am;
(b) at noon, Eugene is at $(3,4)$; two hours earlier Eugene was at $(6,2)$;
(c) during a single hour, a small airplane flew 40 miles north and 100 miles west.

3. A bug is initially at $(-3,7)$. Where is the bug after being displaced by vector $[-7, 8]$?



The diagram above shows the relative locations of Santa Fe, Albuquerque and Phoenix. Label the cities ***S, A***, and ***P***. What do the vectors $\overrightarrow{SA}$, $\overrightarrow{AP}$ and $\overrightarrow{SP}$ represent? What would be a logical way to mathematically describe the relationship between them?

Geometrically Adding Vectors
The sum of two vectors ***a*** and ***b*** is obtained by placing the vectors head to tail and drawing the vector from the available tail to the available head. The sum is sometimes called the ***resultant vector***.

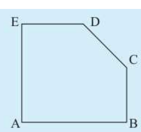
Is ***a*** + ***b*** the same as ***b*** + ***a***? Discuss. What would happen if ***b*** = -***a***?

The ***zero*** vector can be written as (0, 0), [0, 0], $\begin{pmatrix}0\\0\end{pmatrix}$ or $\vec{0}$

Let's try a few:

Find a single vector which is equal to:

a $\overrightarrow{BC} + \overrightarrow{CA}$
b $\overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{EC}$
c $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$
d $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

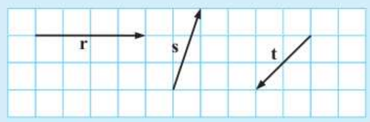


How do you think we should define ***subtraction*** of two vectors?

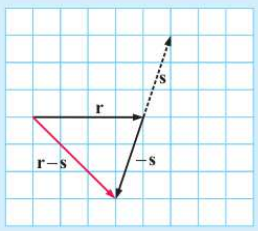
Geometrically Subtracting Vectors
To subtract vector ***b*** from vector ***a*** (***a*** - ***b***) add the opposite of ***b*** to ***a.***

For $\mathbf{r}$, $\mathbf{s}$, and $\mathbf{t}$ shown, find geometrically:

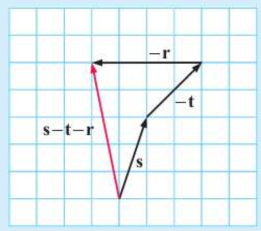
a $\mathbf{r} - \mathbf{s}$
b $\mathbf{s} - \mathbf{t} - \mathbf{r}$



a



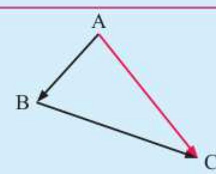
b



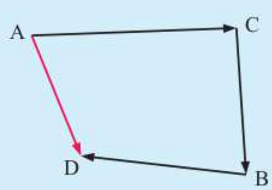
For points A, B, C, and D, simplify the following vector expressions:

a $\overrightarrow{AB} - \overrightarrow{CB}$ **b** $\overrightarrow{AC} - \overrightarrow{BC} - \overrightarrow{DB}$

a $\overrightarrow{AB} - \overrightarrow{CB}$
$= \overrightarrow{AB} + \overrightarrow{BC}$ $\{$as $\overrightarrow{BC} = -\overrightarrow{CB}\}$
$= \overrightarrow{AC}$



b $\overrightarrow{AC} - \overrightarrow{BC} - \overrightarrow{DB}$
$= \overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BD}$
$= \overrightarrow{AD}$



12B.1: #1-4 all (Addition)
12B.2: #1-3 all (Subtraction)