Circular Functions and Trig - Practice Problems (to 07) MarkScheme

1. (a) Evidence of using the cosine rule (M1)

eg $\cos \text{ P}\hat{\text{Q}}\text{R} = \dfrac{p^2+r^2-q^2}{2pr}$, $q^2 = p^2+r^2-2pr\cos \text{ P}\hat{\text{Q}}\text{R}$

Correct substitution A1

eg $\dfrac{4^2+6^2-5^2}{2\times4\times6}$, $5^2 = 4^2+6^2-2\times4\times6\cos Q$

$\cos \text{ P}\hat{\text{Q}}\text{R} = \dfrac{27}{48} = 0.5625$ (A1)

$\text{P}\hat{\text{Q}}\text{R} = 55.8°$ (0.973 radians) A1N2

(b) Area $= \dfrac{1}{2}pr\sin \text{ P}\hat{\text{Q}}\text{R}$

For substituting correctly $\dfrac{1}{2}\times4\times6 \sin 55.8$ A1

$= 9.92$ (cm^2) A1N1

[6]

2. ***Note***: *Throughout this question, do **not** accept methods which involve finding* θ.

(a) Evidence of correct approach A1

eg $\sin\theta = \dfrac{\text{BC}}{\text{AB}}$, $\text{BC} = \sqrt{3^2-2^2} = \sqrt{5}$

$\sin\theta = \dfrac{\sqrt{5}}{3}$ AG N0

(b) Evidence of using $\sin 2\theta = 2\sin\theta\cos\theta$ (M1)

$= 2\left(\dfrac{\sqrt{5}}{3}\right)\left(\dfrac{2}{3}\right)$ A1

$= \dfrac{4\sqrt{5}}{9}$ AGN0

(c) Evidence of using an appropriate formula for $\cos 2\theta$ M1

eg $\dfrac{4}{9}-\dfrac{5}{9}$, $2\times\dfrac{4}{9}-1$, $1-2\times\dfrac{5}{9}$, $\sqrt{\left(1-\dfrac{80}{81}\right)}$

$\cos 2\theta = -\dfrac{1}{9}$ A2 N2

[6]

3. (a) For using perimeter = $r + r$ + arc length (M1)

$20 = 2r + r\theta$ A1

$\theta = \dfrac{20-2r}{r}$ AG N0

(b) Finding $A = \dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)$ $\left(=10r-r^2\right)$ (A1)

For setting up equation in r M1

Correct simplified equation, or sketch

eg $10r - r^2 = 25$, $r^2 - 10r + 25 = 0$ (A1)

$r = 5$ cm A1 N2

[6]

4. ***Notes**: Candidates may have differing answers due to using approximate answers from previous parts or using answers from the GDC. Some leeway is provided to accommodate this.*

(a) **METHOD 1**

Evidence of using the cosine rule (M1)

eg $\cos \boldsymbol{C} = \dfrac{a^2+b^2-c^2}{2ab}, a^2 = b^2+c^2-2bc\cos A$

Correct substitution

eg $\cos \hat{A}\hat{O}P = \dfrac{3^2+2^2-4^2}{2\times 3\times 2}, 4^2 = 3^2+2^2-2\times 3\times 2\cos \hat{A}\hat{O}P$ A1

$\cos \hat{A}\hat{O}P = -0.25$

$\hat{A}\hat{O}P = 1.82 \left(= \dfrac{26\pi}{45}\right)$ (radians) A1 N2

METHOD 2

Area of AOBP = 5.81 (from part (d))

Area of triangle AOP = 2.905 (M1)

$2.9050 = 0.5 \times 2 \times 3 \times \sin \hat{A}\hat{O}P$ A1

$\hat{A}\hat{O}P = 1.32$ or 1.82

$\hat{A}\hat{O}P = 1.82 \left(= \dfrac{26\pi}{45}\right)$ (radians) A1 N2

(b) $\hat{A}\hat{O}B = 2(\pi - 1.82) \quad (= 2\pi - 3.64)$ (A1)

$= 2.64 \left(= \dfrac{38\pi}{45}\right)$ (radians) A1N2

(c) (i) Appropriate method of finding area (M1)

eg area $= \dfrac{1}{2}\theta r^2$

Area of sector PAEB $= \dfrac{1}{2}\times 4^2 \times 1.63$ A1

$= 13.0$ (cm^2)

(accept the exact value 13.04) A1N2

(ii) Area of sector OADB $= \dfrac{1}{2}\times 3^2 \times 2.64$ A1

$= 11.9$ (cm^2) A1N1

(d) (i) Area AOBE = Area PAEB – Area AOBP (= 13.0 – 5.81) M1

= 7.19 (accept 7.23 from the exact answer for PAEB) A1N1

(ii) Area shaded = Area OADB – Area AOBE (= 11.9 – 7.19) M1

= 4.71 (accept answers between 4.63 and 4.72) A1N1

[14]

5. (a) Evidence of choosing cosine rule (M1)

eg $a^2 = b^2 + c^2 - 2bc \cos A$

Correct substitution A1

eg $(AD)^2 = 7.1^2 + 9.2^2 - 2(7.1)(9.2)\cos 60°$

$(AD)^2 = 69.73$ (A1)

AD = 8.35 (cm) A1N2

(b) $180° - 162° = 18°$ (A1)

Evidence of choosing sine rule (M1)

Correct substitution A1

eg $\frac{DE}{\sin 18°} = \frac{8.35}{\sin 110°}$

DE = 2.75 (cm) A1 N2

(c) Setting up equation (M1)

eg $\frac{1}{2}\ ab \sin C = 5.68,\ \frac{1}{2}\ bh = 5.68$

Correct substitution A1

eg $5.68 = \frac{1}{2}(3.2)\ (7.1) \sin\ D\hat{B}C,\ \frac{1}{2} \times 3.2 \times h = 5.68,\ (h = 3.55)$

$\sin\ D\hat{B}C = 0.5$ (A1)

$D\hat{B}C$ 30° and/or 150° A1 N2

(d) Finding $A\hat{B}C$ (60° + $D\hat{B}C$) (A1)

Using appropriate formula (M1)

eg $(AC)^2 = (AB)^2 + (BC)^2$, $(AC)^2 = (AB)^2 + (BC)^2 - 2\ (AB)\ (BC) \cos ABC$

Correct substitution (allow ***FT*** on **their** seen $A\hat{B}C$)

eg $(AC)^2 = 9.2^2 + 3.2^2$ A1

AC = 9.74 (cm) A1 N3

(e) For finding area of triangle ABD (M1)

Correct substitution Area = $\frac{1}{2} \times 9.2 \times 7.1 \sin 60°$ A1

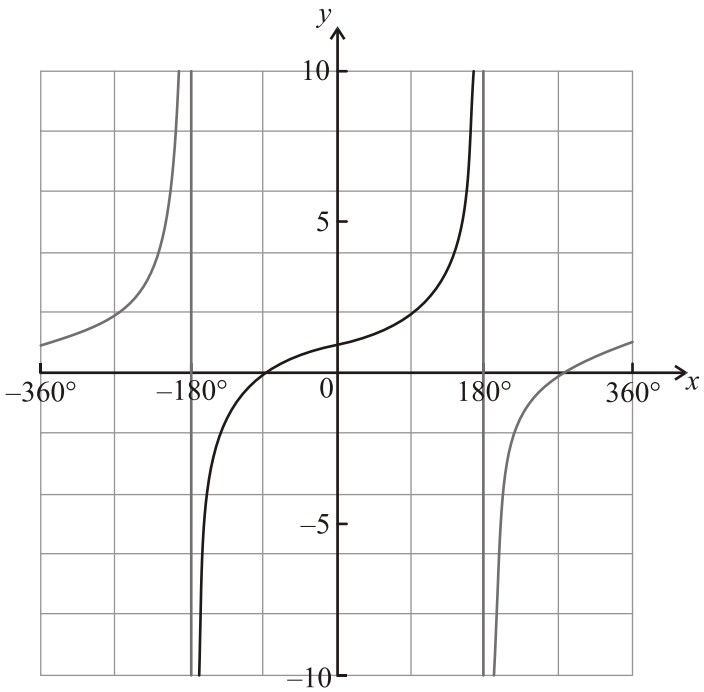
= 28.28... A1

Area of ABCD = 28.28... + 5.68 (M1)

= 34.0 (cm^2) A1N3

[21]

6. (a)



Correct asymptotes A1A1 N2

(b) (i) Period = 360° (accept 2π) A1 N1

(ii) $f(90°) = 2$ A1 N1

(c) 270°, –90° A1A1 N1N1

***Notes**: Penalize **1 mark** for any additional values.*
*Penalize **1 mark** for correct answers given in radians* $\left(\frac{3\pi}{2}, -\frac{\pi}{2}, \text{or } 4.71, -1.57\right)$.

[6]

7. (a) **METHOD 1**

Using the discriminant $\Delta = 0$ (M1)

$k^2 = 4 \times 4 \times 1$

$k = 4, k = -4$ A1A1 N3

METHOD 2

Factorizing (M1)

$(2x \pm 1)^2$

$k = 4, k = -4$ A1A1 N3

(b) Evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ M1

eg $2(2\cos^2\theta - 1) + 4\cos\theta + 3$

$f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$ AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

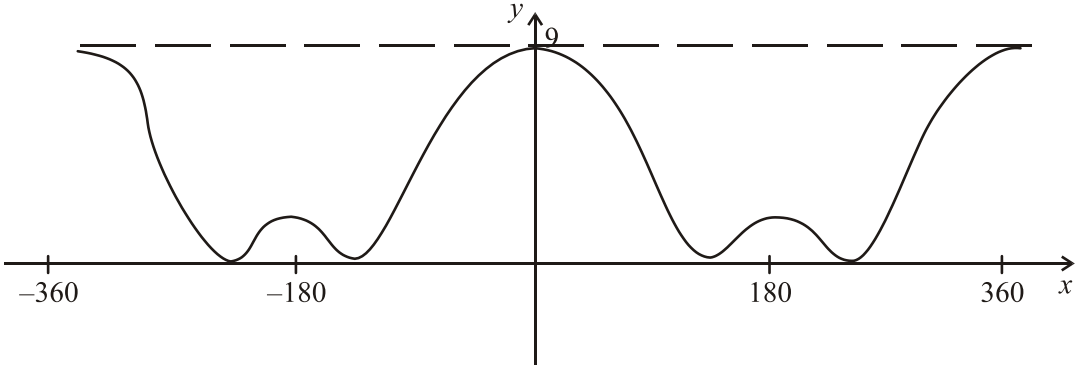
Attempting to solve for $\cos\theta$ M1

$\cos\theta = -\frac{1}{2}$ (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2N3

METHOD 2

Sketch of $y = 4\cos^2\theta + 4\cos\theta + 1$ M1



Indicating 4 zeros (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2N3

(d) Using sketch (M1)

$c = 9$ A1 N2

[11]

8. ***Note:** Accept **exact** answers given in terms of π.*

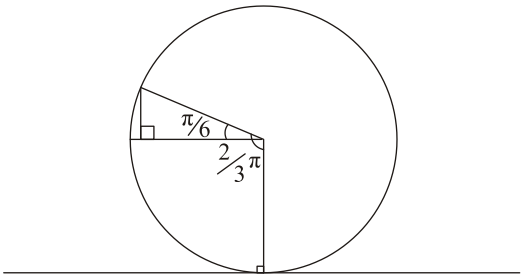
(a) Evidence of using $l = r\theta$ (M1)

arc AB = 7.85 (m) A1 N2

(b) Evidence of using $A = \frac{1}{2}r^2\theta$ (M1)

Area of sector AOB = 58.9 (m^2) A1 N2

(c) **METHOD 1**



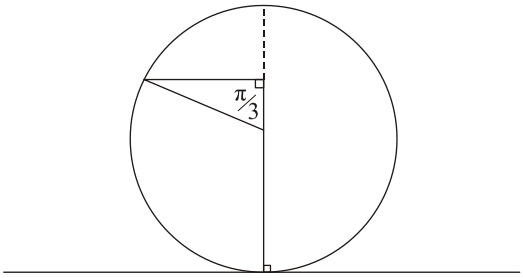
angle = $\frac{\pi}{6}\ (30°)$ (A1)

attempt to find $15 \sin \frac{\pi}{6}$ M1

height = $15 + 15 \sin \frac{\pi}{6}$

= 22.5 (m) A1 N2

METHOD 2



angle = $\frac{\pi}{3}\ (60°)$ (A1)

attempt to find $15 \cos \frac{\pi}{3}$ M1

height = $15 + 15 \cos \frac{\pi}{3}$

= 22.5 (m) A1 N2

(d) (i) $h\left(\frac{\pi}{4}\right) = 15 - 15\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ (M1)

= 25.6 (m) A1N2

(ii) $h(0) = 15 - 15 \cos\left(0 + \frac{\pi}{4}\right)$ (M1)

= 4.39(m) A1N2

(iii) **METHOD 1**

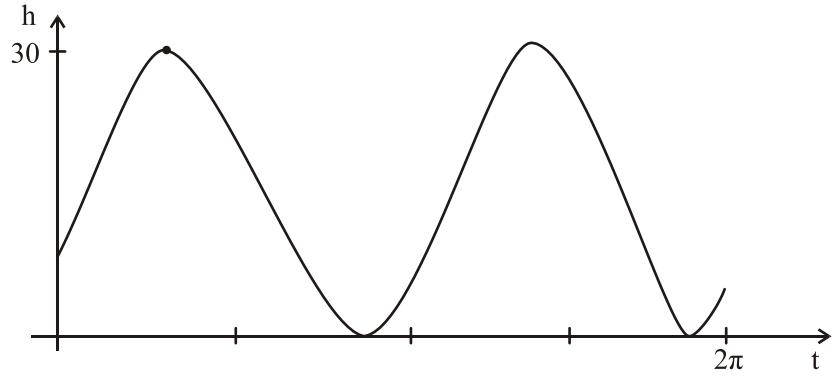
Highest point when $h = 30$ R1

$30 = 15 - 15 \cos\left(2t + \frac{\pi}{4}\right)$ M1

$\cos\left(2t + \frac{\pi}{4}\right) = -1$ (A1)

$t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$ A1N2

METHOD 2



Sketch of graph of h M2
Correct maximum indicated (A1)
$t = 1.18$ A1N2

METHOD 3

Evidence of setting $h'(t) = 0$ M1

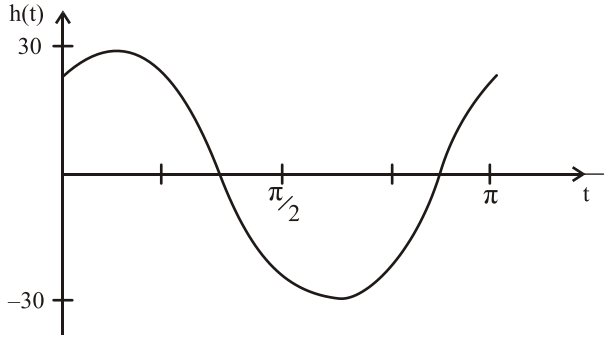
$\sin\left(2t+\frac{\pi}{4}\right)=0$ (A1)

Justification of maximum R1
eg reasoning from diagram, first derivative test, second derivative test

$t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$ A1N2

(e) $h'(t) = 30\sin\left(2t+\frac{\pi}{4}\right)$ (may be seen in part (d)) A1A1 N2

(f) (i)



A1A1A1N3

Notes: *Award A1 for range –30 to 30, A1 for two zeros.*
Award A1 for approximate correct ***sinusoidal*** *shape.*

(ii) **METHOD 1**

Maximum on graph of h' (M1)
$t = 0.393$ A1N2

METHOD 2

Minimum on graph of h' (M1)
$t = 1.96$ A1N2

METHOD 3

Solving $h''(t) = 0$ (M1)
One or both correct answers A1
$t = 0.393$, $t = 1.96$ N2

[22]

9. (a) Vertex is (4, 8) A1A1 N2

(b) Substituting $-10 = a(7-4)^2 + 8$ M1

$a = -2$ A1N1

(c) For y-intercept, $x = 0$ (A1)

$y = -24$ A1 N2

[6]

10. **METHOD 1**

Evidence of correctly substituting into $A = \frac{1}{2}r^2\theta$ A1

Evidence of correctly substituting into $l = r\theta$ A1

For attempting to eliminate one variable … (M1)

leading to a correct equation in one variable A1

$r = 4 \quad \theta = \frac{\pi}{6} \quad (= 0.524, 30°)$ A1A1 N3

METHOD 2

Setting up and equating ratios (M1)

$\frac{\frac{4}{3}\pi}{\pi r^2} = \frac{\frac{2}{3}\pi}{2\pi r}$ A1A1

Solving gives $r = 4$ A1

$r\theta = \frac{2}{3}\pi \left(\text{or } \frac{1}{2}r^2\theta = \frac{4}{3}\pi \right)$ A1

$\theta = \frac{\pi}{6} \ (= 0.524, 30°)$ A1

$r = 4 \quad \theta = \frac{\pi}{6} \ (= 0.524, 30°)$ N3

[6]

11. $a = 4, b = 2, c = \frac{\pi}{2} \left(\text{or } \frac{3\pi}{2} \text{ etc} \right)$ A2A2A2 N6

[6]

12. (a) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1A1 N2

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A2A1A1 N4

[6]

13. **METHOD 1**

Evidence of correctly substituting into $l = r\theta$ A1

Evidence of correctly substituting into $A = \frac{1}{2}r^2\theta$ A1

For attempting to solve these equations (M1)

eliminating one variable correctly A1

$r = 15 \quad \theta = 1.6 \quad (= 91.7°)$ A1A1 N3

METHOD 2

Setting up and equating ratios (M1)

$\frac{24}{2\pi r}=\frac{180}{\pi r^2}$ A1A1

Solving gives $r = 15$ A1

$r\theta = 24 \quad \left(\text{or}\ \frac{1}{2}r^2\theta = 180\right)$ A1

$\theta = 1.6 \quad (= 91.7°)$ A1

$r = 15 \quad \theta = 1.6 \quad (= 91.7°)$ N3

[6]

14. (a) For **correct** substitution into cosine rule A1

$BD = \sqrt{4^2 + 8^2 - 2\times 4\times 8\cos\theta}$

For factorizing 16, $BD = \sqrt{16(5 - 4\cos\theta)}$ A1

$= 4\sqrt{5 - 4\cos\theta}$ AGN0

(b) (i) BD = 5.5653 ... (A1)

$\frac{\sin C\hat{B}D}{12}=\frac{\sin 25}{5.5653}$ M1A1

$\sin C\hat{B}D = 0.911$ (accept 0.910, subject to ***AP***) A1N2

(ii) $C\hat{B}D = 65.7°$ A1 N1

Or $C\hat{B}D = 180 -$ their acute angle (M1)

$= 114°$ A1N2

(iii) $B\hat{D}C = 89.3°$ (A1)

$\frac{BC}{\sin 89.3}=\frac{5.5653}{\sin 25}$ or $\frac{BC}{\sin 89.3}=\frac{12}{\sin 65.7}$ (or cosine rule) M1A1

$BC = 13.2$ (accept 13.17…) A1

Perimeter = 4 + 8 + 12 + 13.2

$= 37.2$ A1N2

(c) Area $= \frac{1}{2}\times 4\times 8\times \sin 40°$ A1

$= 10.3$ A1 N1

[16]

15. (a) **METHOD 1**

Note: *There are many valid algebraic approaches to this problem (eg completing the square, using* $x = \frac{-b}{2a}$*). Use the following mark allocation as a guide.*

(i) Using $\frac{dy}{dx} = 0$ (M1)

$-32x + 160 = 0$ A1

$x = 5$ A1N2

(ii) $y_{max} = -16(5^2) + 160(5) - 256$

$y_{max} = 144$ A1N1

METHOD 2

(i) Sketch of the correct parabola (may be seen in part (ii)) M1

$x = 5$ A2N2

(ii) $y_{max} = 144$ A1 N1

(b) (i) $z = 10 - x$ (accept $x + z = 10$) A1 N1

(ii) $z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$ A2 N2

(iii) Substituting for z into the expression in part (ii) (M1)

Expanding $100 - 20x + x^2 = x^2 + 36 - 12x \cos Z$ A1

Simplifying $12x \cos Z = 20x - 64$ A1

Isolating $\cos Z = \frac{20x - 64}{12x}$ A1

$\cos Z = \frac{5x-16}{3x}$ AGN0

Note: *Expanding, simplifying and isolating may be done in any order, with the final A1 being awarded for an expression that clearly leads to the required answer.*

(c) Evidence of using the formula for area of a triangle

$\left(A = \frac{1}{2} \times 6 \times x \times \sin Z\right)$ M1

$A = 3x \sin Z \left(A^2 = \frac{1}{4} \times 36x^2 \times \sin^2 Z\right)$ A1

$A^2 = 9x^2 \sin^2 Z$ AG N0

(d) Using $\sin^2 Z = 1 - \cos^2 Z$ (A1)

Substituting $\frac{5x-16}{3x}$ for $\cos Z$ A1

for expanding $\left(\frac{5x-16}{3x}\right)^2$ to $\left(\frac{25x^2-160x+256}{9x^2}\right)$ A1

for simplifying to an expression that clearly leads to the required answer A1

eg $A^2 = 9x^2 - (25x^2 - 160x + 256)$

$A^2 = -16x^2 + 160x - 256$ AG

(e) (i) 144 (is maximum value of A^2, from part (a)) A1

$A_{max} = 12$ A1N1

(ii) Isosceles A1 N1

[20]

16. (a) Evidence of choosing the double angle formula (M1)

$f(x) = 15 \sin (6x)$ A1 N2

(b) Evidence of substituting for $f(x)$ (M1)

eg $15 \sin 6x = 0$, $\sin 3x = 0$ **and** $\cos 3x = 0$

$6x = 0, \pi, 2\pi$

$x = 0, \frac{\pi}{6}, \frac{\pi}{3}$ A1A1A1 N4

[6]

17. (a) (i) OP = PQ (= 3cm) R1

So Δ OPQ is isosceles AGN0

(ii) Using cos rule correctly *eg* $\cos \hat{OPQ} = \frac{3^2 + 3^2 - 4^2}{2 \times 3 \times 3}$ (M1)

$\cos \hat{OPQ} = \frac{9+9-16}{18} \left(= \frac{2}{18}\right)$ A1

$\cos \hat{OPQ} = \frac{1}{9}$ AGN0

(iii) Evidence of using $\sin^2 A + \cos^2 A = 1$ M1

$\sin \hat{OPQ} = \sqrt{1-\frac{1}{81}} \left(=\sqrt{\frac{80}{81}}\right)$ A1

$\sin \hat{OPQ} = \frac{\sqrt{80}}{9}$ AGN0

(iv) Evidence of using area triangle OPQ = $\frac{1}{2}\times OP\times PQ \sin P$ M1

eg $\frac{1}{2}3\times3\frac{\sqrt{80}}{9}, \frac{9}{2}\times0.9938\ldots$

Area triangle OPQ = $\frac{\sqrt{80}}{2}$ $\left(=\sqrt{20}\right)(=4.47)$ A1N1

(b) (i) $\hat{OPQ} = 1.4594...$

$\hat{OPQ} = 1.46$ A1N1

(ii) Evidence of using formula for area of a sector (M1)

eg Area sector OPQ = $\frac{1}{2}\times3^2\times1.4594\ldots$

$= 6.57$ A1N2

(c) $\hat{QOP} = \frac{\pi-1.4594\ldots}{2}(=0.841)$ (A1)

Area sector QOS = $\frac{1}{2}\times4^2\times0.841$ A1

$= 6.73$ A1N2

(d) Area of small semi-circle is 4.5π (= 14.137...) A1

Evidence of correct approach M1

eg Area = area of semi-circle – area sector OPQ – area sector QOS + area triangle POQ

Correct expression A1

eg $4.5\pi - 6.5675... - 6.7285... + 4.472...$, $4.5\pi - (6.7285... + 2.095...)$, $4.5\pi - (6.5675... + 2.256...)$

Area of the shaded region = 5.31 A1 N1

[17]

18. (a) $p = 30$ A22

(b) **METHOD 1**

Period = $\frac{2\pi}{q}$ (M2)

$= \frac{\pi}{2}$ (A1)

$\Rightarrow q = 4$ A1 4

METHOD 2

Horizontal stretch of scale factor = $\frac{1}{q}$ (M2)

scale factor = $\frac{1}{4}$ (A1)

$\Rightarrow q = 4$ A1 4

[6]

19. (a) using the cosine rule *(A2)* = $b^2 + c^2 - 2bc \cos \hat{A}$ (M1)

substituting correctly $BC^2 = 65^2 + 104^2 - 2\,(65)\,(104) \cos 60°$ A1

$= 4225 + 10816 - 6760 = 8281$

$\Rightarrow$ BC = 91 m A13

(b) finding the area, using $\frac{1}{2}bc\sin\hat{A}$ (M1)

substituting correctly, area = $\frac{1}{2}(65)(104)\sin 60°$ A1

$= 1690\sqrt{3}$ (Accept $p = 1690$) A1 3

(c) (i) $A_1 = \left(\frac{1}{2}\right)(65)(x)\sin 30°$ A1

$= \frac{65x}{4}$ AG 1

(ii) $A_2 = \left(\frac{1}{2}\right)(104)(x)\sin 30°$ M1

$= 26x$ A1 2

(iii) starting $A_1 + A_2 = A$ or substituting $\frac{65x}{4} + 26x = 1690\sqrt{3}$ (M1)

simplifying $\frac{169x}{4} = 1690\sqrt{3}$ A1

$x = \frac{4\times 1690\sqrt{3}}{169}$ A1

$\Rightarrow x = 40\sqrt{3}$ (Accept $q = 40$) A1 4

(d) (i) Recognizing that supplementary angles have equal sines

eg $A\hat{D}C = 180 - A\hat{D}B \Rightarrow \sin A\hat{D}C = \sin A\hat{D}B$ R1

(ii) using sin rule in ΔADB and ΔACD (M1)

substituting correctly $\frac{BD}{\sin 30°} = \frac{65}{\sin A\hat{D}B} \Rightarrow \frac{BD}{65} = \frac{\sin 30°}{\sin A\hat{D}B}$ A1

and $\frac{DC}{\sin 30°} = \frac{104}{\sin A\hat{D}B} \Rightarrow \frac{DC}{104} = \frac{\sin 30°}{\sin A\hat{D}C}$ M1

since $\sin A\hat{D}B = \sin A\hat{D}C$

$\frac{BD}{65} = \frac{DC}{104} \Rightarrow \frac{BD}{DC} = \frac{65}{104}$ A1

$\Rightarrow \frac{BD}{DC} = \frac{5}{8}$ AG 5

[18]

20. (a) $A = \frac{1}{2}r^2\theta$

$27 = \frac{1}{2}(1.5)r^2$ (M1)(A1)

$r^2 = 36$ (A1)

$r = 6$ cm (A1) (C4)

(b) Arc length $= r\theta = 1.5\times 6$ (M1)

Arc length = 9 cm (A1) (C2)

Note: *Penalize a total of (1 mark) for missing units.*

[6]

21. (a) when $y = 0$ (may be implied by a sketch) (A1)

$x = \frac{8\pi}{9}$ or 2.79 (A1) (C2)

(b) **METHOD 1**

Sketch of appropriate graph(s) (M1)
Indicating **correct** points (A1)
$x = 3.32$ or $x = 5.41$ (A1)(A1)(C2)(C2)

METHOD 2

$\sin\left(x + \frac{\pi}{9}\right) = -\frac{1}{2}$

$x + \frac{\pi}{9} = \frac{7\pi}{6}$, $x + \frac{\pi}{9} = \frac{11\pi}{6}$ (A1)(A1)

$x = \frac{7\pi}{6} - \frac{\pi}{9}$, $x = \frac{11\pi}{6} - \frac{\pi}{9}$

$x = \frac{19\pi}{18}$, $x = \frac{31\pi}{18}$ $(x = 3.32,\ x = 5.41)$ (A1)(A1)(C2)(C2)

[6]

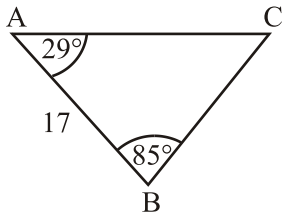
22. (a) for using cosine rule $\left(a^2 = b^2 + c^2 - 2ab\cos C\right)$ (M1)

$BC^2 = 15^2 + 17^2 - 2(15)(17)\cos 29^\circ$ (A1)

$BC = 8.24$ m (A1) (N0) 3

Notes: *Either the first or the second line may be implied, but not both. Award* ***no marks*** *if 8.24 is obtained by assuming a right (angled) triangle (BC = 17 sin 29).*

(i)



$A\hat{C}B = 180 - (29 + 85) = 66^\circ$

for using sine rule (may be implied) (M1)

$\frac{AC}{\sin 85^\circ} = \frac{17}{\sin 66^\circ}$ (A1)

$AC = \frac{17\sin 85^\circ}{\sin 66^\circ}$

$AC = (18.5380\ldots) = 18.5$ m (A1) (N2)

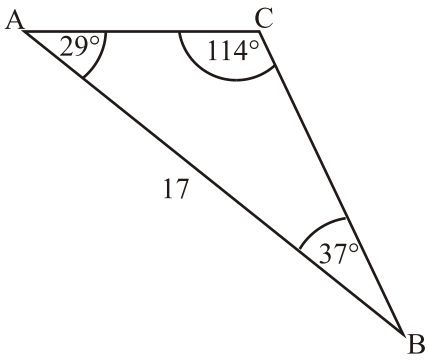
(ii) $\text{Area} = \frac{1}{2}(17)(18.538\ldots)\sin 29^\circ$ (A1)

$= 76.4\ \text{m}^2$ (Accept $76.2\ \text{m}^2$) (A1) (N1) 5

(c) $A\hat{C}B$ from previous triangle $= 66^\circ$

Therefore alternative $A\hat{C}B = 180 - 66 = 114^\circ$ (or $29 + 85$) (A1)

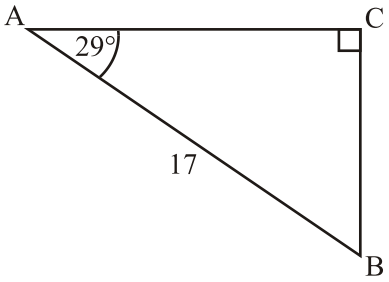
$A\hat{B}C = 180 - (29 + 114) = 37^\circ$



$\frac{AC}{\sin 37^\circ} = \frac{17}{\sin 114^\circ}$ (M1)(A1)

$AC = (11.19906\ldots) = 11.2$ m (A1) (N1) 4

(d)



Minimum length for BC when $A\hat{C}B = 90°$ or diagram showing right triangle (M1)

$\sin 29^\circ = \frac{CB}{17}$

$CB = 17\sin 29^\circ$

$CB = (8.2417\ldots) = 8.24$ m (A1) (N1) 2

[14]

23. (a) (i) $f'(x) = \frac{1}{2} \times 2\cos 2x - \sin x$

$= \cos 2x - \sin x$ (A1)(A1) (N2)

Note: *Award (A1)(A1) for* $-2\sin^2 x - \sin x + 1$ *only if work shown, using product rule on* $\sin x \cos x + \cos x$.

(ii) $2\sin^2 x + \sin x - 1 = (2\sin x - 1)(\sin x + 1)$ or

$2(\sin x - 0.5)(\sin x + 1)$ (A1) (N1)

(iii) $2\sin x = 1$ or $\sin x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} = (0.524) \quad x = \frac{5\pi}{6} = (2.62) \quad x = \frac{3\pi}{2} = (4.71)$ (A1)(A1)(A1)(N1)

(N1)(N1)6

(b) $x = \frac{\pi}{6} (= 0.524)$ (A1) (N1) 1

(c) (i) **EITHER**

curve crosses axis when $x = \frac{\pi}{2}$ (may be implied) (A1)

$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(x)\,dx + \left| \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} f(x)\,dx \right|$ (M1)(A1)(N3)

OR

$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} |f(x)|\,dx$ (M1)(A2)(N3)

(ii) $\text{Area} = 0.875 + 0.875$ (M1)

$= 1.75$ (A1) (N2) 5

[12]

24. Using area of a triangle = $\frac{1}{2}\,ab \sin C$ (M1)

$20 = \frac{1}{2}(10)(8)\sin \boldsymbol{Q}$ (A1)(A1)(A1)

Note: *Accept any letter for Q*

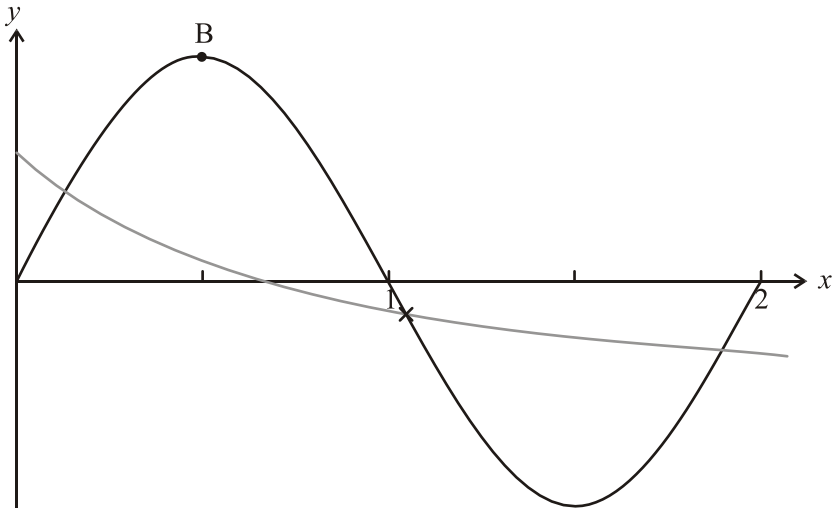
$\sin Q = 0.5$ (A1)

$\text{P}\hat{\text{Q}}\text{R} = 30°$ or $\frac{\pi}{6}$ or 0.524 (A1) (C6)

[6]

25. (a) $b = 6$ (A1) (C1)

(b)



(A3) (C3)

(c) $x = 1.05$ (accept (1.05, −0.896)) (correct answer only, no additional solutions) (A2) (C2)

[6]

26. (a) $3(1 - 2\sin^2 x) + \sin x = 1$ (A1)

$6\sin^2 x - \sin x - 2 = 0$ ($p = 6$, $q = -1$, $r = -2$) (A1) (C2)

(b) $(3\sin x - 2)(2\sin x + 1)$ (A1)(A1) (C2)

(c) 4 solutions (A2) (C2)

[6]

27. Area of large sector $\frac{1}{2}r^2\theta = \frac{1}{2}16^2 \times 1.5$ (M1)

$= 192$ (A1)

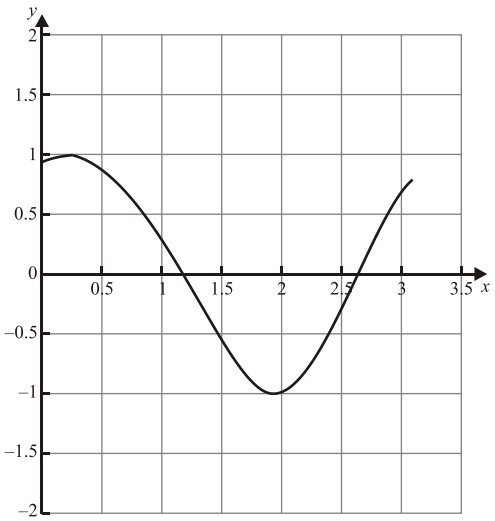
Area of small sector $\frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times 1.5$ (M1)

$= 75$ (A1)

Shaded area = large area – small area = 192 – 75 (M1)
= 117 (A1) (C6)

[6]

28. (a)



(A1)(A1)(C2)

Note: *Award (A1) for the graph crossing the y-axis between 0.5 and 1, and (A1) for an approximate sine curve crossing the x-axis twice. Do* ***not*** *penalize for x >3.14.*

(b) (Maximum) $x = 0.285\ldots \left(\frac{\pi}{4} - \frac{1}{2}\right)$ (A1)

$x = 0.3$ (1 dp) (A1) (C2)

(Minimum) $x = 1.856\ldots \left(\frac{3\pi}{4} - \frac{1}{2}\right)$ (A1)

$x = 1.9$ (1 dp) (A1) (C2)

[6]

29. Area of a triangle = $\frac{1}{2} \times 3 \times 4 \sin A$ (A1)

$\frac{1}{2} \times 3 \times 4 \sin A = 4.5$ (A1)

$\sin A = 0.75$ (A1)

$A = 48.6°$ and $A = 131°$ (or 0.848, 2.29 radians) (A1)(A2) (C6)

Note: *Award (C4) for 48.6° only, (C5) for 131° only.*

[6]

30. **METHOD 1**

$2 \cos^2 x = 2 \sin x \cos x$ (M1)

$2 \cos^2 x - 2 \sin x \cos x = 0$

$2 \cos x(\cos x - \sin x) = 0$ (M1)

$\cos x = 0, (\cos x - \sin x) = 0$ (A1)(A1)

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ (A1)(A1) (C6)

METHOD 2

Graphical solutions

EITHER

for both graphs $y = 2 \cos^2 x$, $y = \sin 2x$, (M2)

OR

for the graph of $y = 2 \cos^2 x - \sin 2x$. (M2)

THEN

Points representing the solutions clearly indicated (A1)

1.57, 0.785 (A1)

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ (A1)(A1) (C6)

Notes: *If no working shown, award (C4) for one correct answer.*
Award (C2)(C2) for each correct decimal answer 1.57, 0.785.
Award (C2)(C2) for each correct degree answer 90°, 45°. Penalize a total of [1 mark] for any additional answers.

[6]

31. (a) (i) $10 + 4 \sin 1 = 13.4$ (A1)

(ii) At 2100, $t = 21$ (A1)

$10 + 4 \sin 10.5 = 6.48$ (A1) (N2) 3

Note: *Award (A0)(A1) if candidates use t = 2100 leading to y = 12.6. No other **ft** allowed.*

(b) (i) 14 metres (A1)

(ii) $14 = 10 + 4 \sin\left(\frac{t}{2}\right) \Rightarrow \sin\left(\frac{t}{2}\right) = 1$ (M1)

$\Rightarrow t = \pi$ (3.14) (correct answer only) (A1) (N2) 3

(c) (i) 4 (A1)

(ii) $10 + 4 \sin\left(\frac{t}{2}\right) = 7$ (M1)

$\Rightarrow \sin\left(\frac{t}{2}\right) = -0.75$ (A1)

$\Rightarrow t = 7.98$ (A1) (N3)

(iii) depth < 7 from 8 –11 = 3 hours (M1)

from 2030 – 2330 = 3 hours (M1)

therefore, total = 6 hours (A1) (N3) 7

[13]

32. (a) Angle $A = 80^\circ$ (A1)

$\frac{AB}{\sin 40^\circ} = \frac{5}{\sin 80^\circ}$ (M1)

$AB = 3.26$ cm (A1) (C3)

(b) Area $= \frac{1}{2} ac \sin B = \frac{1}{2}(5)(3.26)\sin 60^\circ$ (M1)(A1)

$= 7.07$ (accept 7.06) cm^2 (A1) (C3)

Note: *Penalize once in this question for absence of units.*

[6]

33. **METHOD 1**

Area sector OAB $= \frac{1}{2}(5)^2(0.8)$ (M1)

$= 10$ (A1)

$ON = 5\cos 0.8 \quad (= 3.483...)$ (A1)

$AN = 5\sin 0.8 \quad (= 3.586.....)$ (A1)

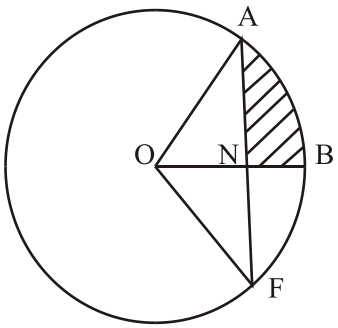
Area of $\Delta AON = \frac{1}{2} ON \times AN$

$= 6.249...$ (cm^2) (A1)

Shaded area $= 10 - 6.249..$

$= 3.75 \text{ (cm}^2\text{)}$ (A1) (C6)

METHOD 2



Area sector $ABF = \frac{1}{2}(5)^2(1.6)$ (M1)

$= 20$ (A1)

Area $\Delta OAF = \frac{1}{2}(5)^2 \sin 1.6$ (M1)

$= 12.5$ (A1)

Twice the shaded area $= 20 - 12.5 \quad (= 7.5)$ (M1)

Shaded area $= \frac{1}{2}(7.5)$

$= 3.75 \text{ (cm}^2\text{)}$ (A1) (C6)

[6]

34. (a) (i) $f'(x) = -6\sin 2x$ (A1)(A1)

(ii) **EITHER**

$f'(x) = -12\sin x \cos x = 0$

$\Rightarrow \sin x = 0$ or $\cos x = 0$ (M1)

OR

$\sin 2x = 0$,

for $0 \le 2x \le 2\pi$ (M1)

THEN

$x = 0, \frac{\pi}{2},$ (A1)(A1)(A1) (N4) 6

(b) (i) translation (A1)

in the y-direction of -1 (A1)

(ii) 1.11 (1.10 from TRACE is subject to **AP**) (A2) 4

[10]

35. $3 = p + q \cos 0$ (M1)

$3 = p + q$ (A1)

$-1 = p + q \cos \pi$ (M1)

$-1 = p - q$ (A1)

(a) $p = 1$ (A1)

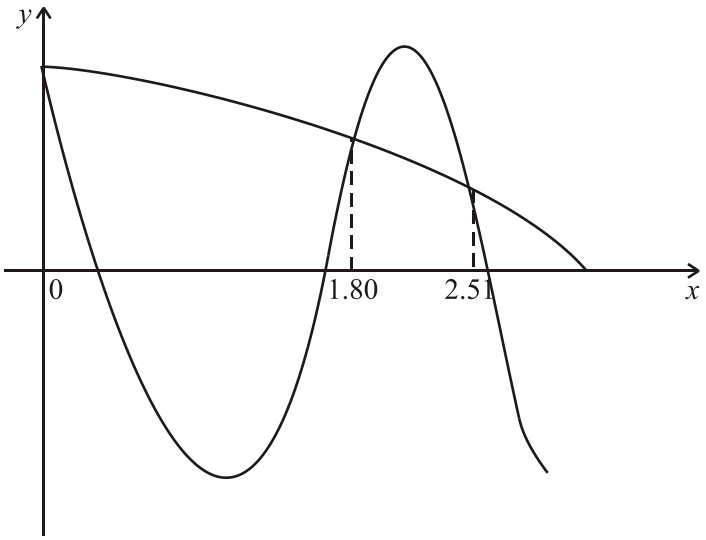
(C3)

(b) $q = 2$ (A1)

(C3)

[6]

36. Method 1



0 (C2)
1.80 [3 sf] (G2) (C2)
2.51 [3 sf] (G2) (C2)

Method 2

$3x = \pm 0.5x + 2\pi$ (*etc.*) (M1)

$\Rightarrow 3.5x = 0, 2\pi, 4\pi$ **or** $2.5x = 0, 2\pi, 4\pi$ (A1)

$7x = 0, 4\pi, (8\pi)$ **or** $5x = 0, 4\pi, (8\pi)$ (A1)

$x = 0, \frac{4\pi}{7}$ **or** $x = 0, \frac{4\pi}{5}$ (A1)(A1)(A1)

$x = 0, \frac{4\pi}{7}, \frac{4\pi}{5}$ (C2)(C2)(C2)

[6]

37. (a) area of sector ABDC $= \frac{1}{4}\pi(2)^2 = \pi$ (A1)

area of segment BDCP = π – area of ΔABC (M1)

$= \pi - 2$ (A1) (C3)

(b) BP $= \sqrt{2}$ (A1)

area of semicircle of radius BP $= \frac{1}{2}\pi(\sqrt{2})^2 = \pi$ (A1)

area of shaded region $= \pi - (\pi - 2) = 2$ (A1)
(C3)

[6]

38. (a) $\overrightarrow{OR} = \overrightarrow{PQ}$

$= \boldsymbol{q} - \boldsymbol{p}$

$= \begin{pmatrix} 10 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ (A1)(A1)

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ (A1) 3

(b) $\cos \hat{OPQ} = \frac{\overrightarrow{PO} \cdot \overrightarrow{PQ}}{\left|\overrightarrow{PO}\right| \times \left|\overrightarrow{PQ}\right|}$ (A1)

$\left|\overrightarrow{PO}\right| = \sqrt{(-7)^2 + (-3)^2} = \sqrt{58}$, $\left|\overrightarrow{PQ}\right| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$ (A1)(A1)

$\overrightarrow{PO} \cdot \overrightarrow{PQ} = -21 + 6 = -15$ (A1)

$\cos \hat{OPQ} = \dfrac{-15}{\sqrt{58}\sqrt{13}} = \dfrac{-15}{\sqrt{754}}$ (AG) 4

(c) (i) Since $\hat{OPQ} + \hat{PQR} = 180°$ (R1)

$\cos \hat{PQR} = -\cos \hat{OPQ} \left(= \dfrac{15}{\sqrt{754}} \right)$ (AG)

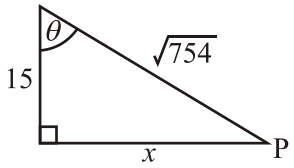
(ii) $\sin \hat{PQR} = \sqrt{1 - \left(\dfrac{15}{\sqrt{754}} \right)^2}$ (M1)

$= \sqrt{\dfrac{529}{754}}$ (A1)

$= \dfrac{23}{\sqrt{754}}$ (AG)

OR

$\cos \theta = \dfrac{15}{\sqrt{754}}$



(M1)

therefore $x^2 = 754 - 225 = 529 \Rightarrow x = 23$ (A1)

$\Rightarrow \sin \theta = \dfrac{23}{\sqrt{754}}$ (AG)

Note: *Award (A1)(A0) for the following solution.*

$\cos \theta = \dfrac{15}{\sqrt{754}} \Rightarrow \theta = 56.89°$

$\Rightarrow \sin \theta = 0.8376$

$\dfrac{23}{\sqrt{754}} = 0.8376 \Rightarrow \sin \theta = \dfrac{23}{\sqrt{754}}$

(iii) Area of OPQR = 2 (area of triangle PQR) (M1)

$= 2 \times \dfrac{1}{2}\left|\overrightarrow{PQ}\right| \times \left|\overrightarrow{QR}\right| \times \sin \hat{PQR}$ (A1)

$= 2 \times \dfrac{1}{2}\sqrt{13}\sqrt{58}\dfrac{23}{\sqrt{754}}$ (A1)

= 23 sq units. (A1)

OR

Area of OPQR = 2 (area of triangle OPQ) (M1)

$= 2\left|\left(\dfrac{1}{2}\right)(7 \times 1 - 3 \times 10)\right|$ (A1)(A1)

= 23 sq units. (A1) 7

Notes: *Other valid methods can be used.*

*Award final (A1) for the **integer** answer.*

[14]

39. (a) Sine rule $\frac{PR}{\sin 35} = \frac{9}{\sin 120}$ (M1)(A1)

$PR = \frac{9\sin 35}{\sin 120}$

$= 5.96$ km (A1) 3

(b) **EITHER**

Sine rule to find PQ

$PQ = \frac{9\sin 25}{\sin 120}$ (M1)(A1)

$= 4.39$ km (A1)

OR

Cosine rule: $PQ^2 = 5.96^2 + 9^2 - (2)(5.96)(9)\cos 25$ (M1)(A1)

$= 19.29$

$PQ = 4.39$ km (A1)

Time for Tom $= \frac{4.39}{8}$ (A1)

Time for Alan $= \frac{5.96}{a}$ (A1)

Then $\frac{4.39}{8} = \frac{5.96}{a}$ (M1)

$a = 10.9$ (A1) 7

(c) $RS^2 = 4QS^2$ (A1)

$4QS^2 = QS^2 + 81 - 18 \times QS \times \cos 35$ (M1)(A1)

$3QS^2 + 14.74QS - 81 = 0$ (**or** $3x^2 + 14.74x - 81 = 0$) (A1)

$\Rightarrow QS = -8.20$ **or** $QS = 3.29$ (G1)

therefore $QS = 3.29$ (A1)

OR

$\frac{QS}{\sin S\hat{R}Q} = \frac{2QS}{\sin 35}$ (M1)

$\Rightarrow \sin S\hat{R}Q = \frac{1}{2}\sin 35$ (A1)

$S\hat{R}Q = 16.7°$ (A1)

Therefore, $Q\hat{S}R = 180 - (35 + 16.7)$

$= 128.3°$ (A1)

$\frac{9}{\sin 128.3} = \frac{QS}{\sin 16.7}\left(= \frac{SR}{\sin 35}\right)$ (M1)

$QS = \frac{9\sin 16.7}{\sin 128.3}\left(= \frac{9\sin 35}{2\sin 128.3}\right)$

$= 3.29$ (A1) 6

[16]

40. (a) (i) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right)+\sin\left(-\frac{\pi}{4}\right)=0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

Note: *Award (A0) for 2.36.*

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = e^x(\cos x + \sin x)$

$\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2e^x \cos x$

(c) $\frac{dy}{dx} = 0$ for a turning point $\Rightarrow 2e^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = e^{\frac{\pi}{2}}(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}) = e^{\frac{\pi}{2}}$

$b = e^{\frac{\pi}{2}}$ (A1) 4

Note: *Award (M1)(A1)(A0)(A0) for a = 1.57, b = 4.81.*

(d) At D, $\frac{d^2y}{dx^2} = 0$ (M1)

$2e^x \cos x - 2e^x \sin x = 0$ (A1)

$2e^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos\frac{\pi}{4} + \sin\frac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area = $\int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

= 7.46 sq units (G1)

OR

Area = 7.46 sq units (G2) 2

Note*: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.*

[17]

41. (a) (i) A is $\left(\frac{4}{3}, 0\right)$ (A1)(A1)(C2)

(ii) B is (0, –4) (A1)(A1)

(C2)

Note: *In each of parts (i) and (ii), award C1 if A and B are interchanged, C1 if intercepts given instead of coordinates.*

(b) Area $= \frac{1}{2} \times 4 \times \frac{4}{3}$ (M1)

$= \frac{8}{3} (= 2.67)$ (A1)

(C2)

[6]

42. (a) $(3 \sin x - 2)(\sin x - 3)$ (A1)(A1)(C2)

Note: *Award A1 if $3x^2 - 11x + 6$ correctly factorized to give $(3x - 2)(x - 3)$ (or equivalent with another letter).*

(b) (i) $(3 \sin x - 2)(\sin x - 3) = 0$

$\sin x = \frac{2}{3}$ $\sin x = 3$ (A1)(A1)

(C2)

(ii) $x = 41.8°, 138°$ (A1)(A1)

(C2)

Notes: *Penalize [1 mark] for any extra answers and [1 mark] for answers in radians.*
ie Award A1 A0 for 41.8°, 138° and any extra answers.
Award A1 A0 for 0.730, 2.41.
Award A0 A0 for 0.730, 2.41 and any extra answers.

[6]

43. ***Note:*** *Do not penalize missing units in this question.*

(a) $AB^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 75°$ (A1)

$= 12^2(2 - 2 \cos 75°)$ (A1)

$= 12^2 \times 2(1 \cos 75°)$

$AB = 12\sqrt{2(1 - \cos 75°)}$ (AG) 2

Note: *The second (A1) is for transforming the initial expression to any simplified expression from which the given result can be clearly seen.*

(b) $P\hat{O}B = 37.5°$ (A1)

$BP = 12 \tan 37.5°$ (M1)

$= 9.21$ cm (A1)

OR

$B\hat{P}A = 105°$ $B\hat{A}P = 37.5°$ (A1)

$\frac{AB}{\sin 105°} = \frac{BP}{\sin 37.5°}$ (M1)

$BP = \frac{AB \sin 37.5°}{\sin 105°} = 9.21(\text{cm})$ (A1) 3

(c) (i) Area $\Delta OBP = \frac{1}{2} \times 12 \times 9.21 \left(\text{or } \frac{1}{2} \times 12 \times 12 \tan 37.5° \right)$ (M1)

$= 55.3$ (cm^2) (accept 55.2 cm^2) (A1)

(ii) Area $\Delta ABP = \frac{1}{2}(9.21)^2 \sin 105°$ (M1)

$= 41.0$ (cm^2) (accept 40.9 cm^2) (A1) 4

(d) Area of sector $= \frac{1}{2}\times 12^2 \times 75 \times \frac{\pi}{180}\left(\text{or } \frac{75}{360}\times\pi\times 12^2\right)$ (M1)

$= 94.2$ (cm^2) (accept 30π or 94.3 (cm^2)) (A1) 2

(e) Shaded area = 2 × area ΔOPB – area sector (M1)

$= 16.4$ (cm^2) (accept 16.2 cm^2, 16.3 cm^2) (A1) 2

[13]

44. ***Note**: Do not penalize missing units in this question.*

(a) (i) At release(P), $t = 0$ (M1)

$s = 48 + 10 \cos 0$

$= 58$ cm below ceiling (A1)

(ii) $58 = 48 + 10 \cos 2\pi t$ (M1)

$\cos 2\pi t = 1$ (A1)

$t = 1$sec (A1)

OR

$t = 1$sec (G3) 5

(b) (i) $\frac{ds}{dt} = -20\pi \sin 2\pi t$ (A1)(A1)

***Note:** Award (A1) for -20π, and (A1) for $\sin 2\pi t$.*

(ii) $v = \frac{ds}{dt} = -20\pi \sin 2\pi t = 0$ (M1)

$\sin 2\pi t = 0$

$t = 0, \frac{1}{2}$ ... (at least 2 values) (A1)

$s = 48 + 10 \cos 0$ **or** $s = 48 + 10 \cos \pi$ (M1)

$= 58$ cm (at P) $= 38$ cm (20 cm above P) (A1)(A1) 7

***Note:** Accept these answers without working for full marks. May be deduced from recognizing that amplitude is 10.*

(c) $48 + 10 \cos 2\pi t = 60 + 15 \cos 4\pi t$ (M1)

$t = 0.162$ secs(A1)

OR

$t = 0.162$ secs (G2) 2

(d) 12 times (G2) 2

***Note:** If either of the correct answers to parts (c) and (d) are missing and suitable graphs have been sketched, award (G2) for sketch of suitable graph(s); (A1) for t = 0.162; (A1) for 12.*

[16]

45. (a) $l = r\theta$ or ACB = 2 × OA (M1)

$= 30$ cm (A1) (C2)

(b) $A\hat{O}B$ (obtuse) $= 2\pi - 2$ (A1)

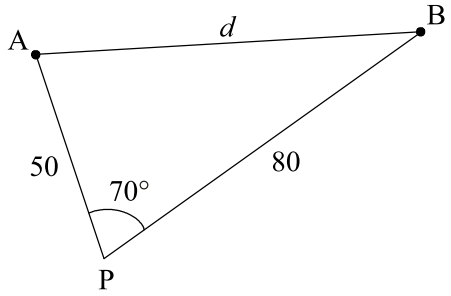
Area $= \frac{1}{2}\theta r^2 = \frac{1}{2}(2\pi - 2)(15)^2$ (M1)(A1)

$= 482$ cm^2 (3 sf) (A1)

(C4)

[6]

46.



(M1)(A2)

OR

$2.5 \times 20 = 50$ (M1)(A1)

$2.5 \times 32 = 80$ (A1)

$d^2 = 50^2 + 80^2 - 2 \times 50 \times 80 \times \cos 70°$ (M1)(A1)

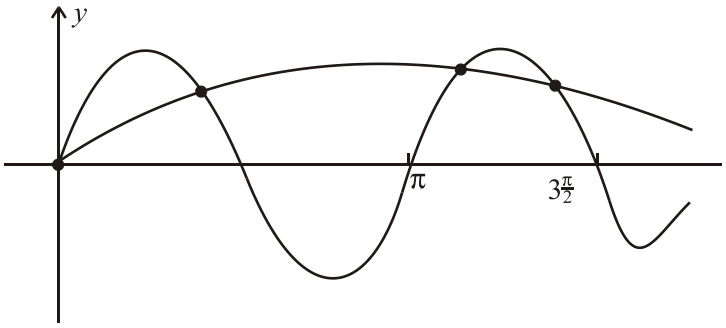
$d = 78.5$ km (A1) (C6)

[6]

47. (a) (i) –1 (A1) (C1)

(ii) 4π (accept 720°) (A2) (C2)

(b)



(G1)

number of solutions: 4 (A2) (C3)

[6]

48.

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example
A	No	$x = -1$ $(\log_{10} 0.1 = -1)$
B	No	$x = 0$ $(\cos 0 = 1)$
C	Yes	N/A

(a) (A3) (C3)

(b) (A3) (C3)

***Notes**: (a) Award (A1) for each correct answer.*

(b) Award (A) marks for statements A and B only if NO in column (a).

Award (A2) for a correct counter example to statement A, (A1) for a correct counter example to statement B (ignore other incorrect examples).

Special Case for statement C:

Award (A1) if candidates write NO, and give a valid reason (eg arctan 1 = $\frac{5\pi}{4}$).

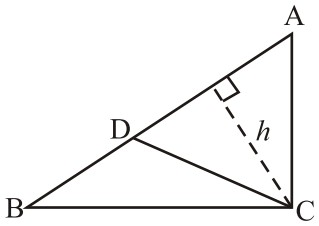
[6]

49. (a) $\dfrac{6}{\sin A} = \dfrac{7\sqrt{2}}{\sin 45°}$ (M1)

$\sin A = 6 \times \dfrac{\sqrt{2}}{2} \times \dfrac{2}{7\sqrt{2}}$ (A1)

$= \dfrac{6}{7}$ (AG) 2

(b)



(i) $B\hat{D}C + B\hat{A}C = 180°$ (A1)

(ii) $\sin A = \dfrac{6}{7}$

$\Rightarrow A = 59.0°$ or $121°$ (3 sf) (A1)(A1)

$\Rightarrow B\hat{C}D = 180° - (121° + 45°)$

$= 14.0°$ (3 sf) (A1)

(iii) $\dfrac{BD}{\sin 14°} = \dfrac{7\sqrt{2}}{\sin 45°}$ (M1)

$\Rightarrow BD = 1.69$ (A1) 6

(c) $\dfrac{\text{Area } \Delta BDC}{\text{Area } \Delta BAC} = \dfrac{\frac{1}{2} \times BD \times h}{\frac{1}{2} \times BA \times h}$ (M1)(A1)

$= \dfrac{BD}{BA}$ (AG) 2

OR

$\dfrac{\text{Area } \Delta BCD}{\text{Area } \Delta BAC} = \dfrac{\frac{1}{2} BD \times 6 \sin 45}{\frac{1}{2} BA \times 6 \sin 45}$ (M1)(A1)

$= \dfrac{BD}{BA}$ (AG) 2

[10]

50. Using sine rule: $\dfrac{\sin B}{5} = \dfrac{\sin 48°}{7}$ (M1)(A1)

$\Rightarrow \sin B = \dfrac{5}{7} \sin 48° = 0.5308\ldots$ (M1)

$\Rightarrow B = \arcsin(0.5308) = 32.06°$ (M1)(A1)

$= 32°$ (nearest degree) (A1) (C6)

Note: *Award a maximum of [5 marks] if candidates give the answer in radians (0.560).*

[6]

51. (a) x is an acute angle => $\cos x$ is positive. (M1)

$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos x = \sqrt{1 - \sin^2 x}$ (M1)

$\Rightarrow \cos x = \sqrt{1 - \left(\frac{1}{3}\right)^2}$ (A1)

$= \sqrt{\frac{8}{9}} \quad \left(= \frac{2\sqrt{2}}{3}\right)$ (A1) (C4)

(b) $\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(\frac{1}{3}\right)^2$ (M1)

$= \frac{7}{9}$ (A1)

(C2)

Notes: *(a) Award (M1)(M0)(A1)(A0) for* $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = 0.943$.

(b) Award (M1)(A0) for $\cos\left(2\sin^{-1}\left(\frac{1}{3}\right)\right) = 0.778$.

[6]

52. (a) $2\sin^2 x = 2(1 - \cos^2 x) = 2 - 2\cos^2 x = 1 + \cos x$ (M1)

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0$ (A1) (C2)

Note: *Award the first (M1) for replacing* $\sin^2 x$ *by* $1 - \cos^2 x$.

(b) $2\cos^2 x + \cos x - 1 = (2\cos x - 1)(\cos x + 1)$ (A1)

(C1)

(c) $\cos x = \frac{1}{2}$ or $\cos x = -1$

$\Rightarrow x = 60°, 180°$ or $300°$ (A1)(A1)(A1)

(C3)

Note: *Award (A1)(A1)(A0) if the correct answers are given in radians (ie* $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$, *or 1.05, 3.14, 5.24)*

[6]

53. (a) The smallest angle is opposite the smallest side.

$\cos\theta = \frac{8^2 + 7^2 - 5^2}{2 \times 8 \times 7}$ (M1)

$= \frac{88}{112} = \frac{11}{14} = 0.7857$

Therefore, $\theta = 38.2°$ (A1) (C2)

(b) Area $= \frac{1}{2} \times 8 \times 7 \times \sin 38.2°$ (M1)

$= 17.3 \text{ cm}^2$ (A1) (C2)

[4]

54. (a) $3\sin^2 x + 4\cos x = 3(1 - \cos^2 x) + 4\cos x$

$= 3 - 3\cos^2 + 4\cos x$ (A1) (C1)

(b) $3\sin^2 x + 4\cos x - 4 = 0 \Rightarrow 3 - 3\cos^2 x + 4\cos x - 4 = 0$

$\Rightarrow 3\cos^2 x - 4\cos x + 1 = 0$ (A1)

$(3\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{3}$ or $\cos x = 1$

$x = 70.5°$ or $x = 0°$ (A1)(A1) (C3)

Note: *Award (C1) for each correct radian answer, ie x = 1.23 or x = 0.*

[4]

55. $\hat{OTA} = 90°$ (A1)

$AT = \sqrt{12^2 - 6^2}$

$= 6\sqrt{3}$

$\hat{TOA} = 60° = \frac{\pi}{3}$ (A1)

Area = area of triangle – area of sector

$= \frac{1}{2} \times 6 \times 6\sqrt{3} - \frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ (M1)

$= 12.3 \text{ cm}^2$ (or $18\sqrt{3} - 6\pi$) (A1) (C4)

OR

$\hat{TOA} = 60°$ (A1)

Area of $\Delta = \frac{1}{2} \times 6 \times 12 \times \sin 60$ (A1)

Area of sector $= \frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ (A1)

Shaded area $= 18\sqrt{3} - 6\pi = 12.3 \text{ cm}^2$ (3 sf) (A1) (C4)

[4]

56. (a) (i) $AP = \sqrt{(x-8)^2 + (10-6)^2} = \sqrt{x^2 - 16x + 80}$ (M1) (AG)

(ii) $OP = \sqrt{(x-0)^2 + (10-0)^2} = \sqrt{x^2 + 100}$ (A1) 2

(b) $\cos \hat{OPA} = \frac{AP^2 + OP^2 - OA^2}{2AP \times OP}$ (M1)

$= \frac{(x^2 - 16x + 80) + (x^2 + 100) - (8^2 + 6^2)}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ (M1)

$= \frac{2x^2 - 16x + 80}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ (M1)

$\cos \hat{OPA} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$ (AG) 3

(c) For $x = 8$, $\cos \hat{OPA} = 0.780869$ (M1)

$\arccos 0.780869 = 38.7°$ (3 sf) (A1)

OR

$\tan \hat{OPA} = \frac{8}{10}$ (M1)

$\hat{OPA} = \arctan(0.8) = 38.7°$ (3 sf) (A1) 2

(d) $\hat{OPA} = 60° \Rightarrow \cos \hat{OPA} = 0.5$

$0.5 = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$ (M1)

$2x^2 - 16x + 80 - \sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}} = 0$ (M1)

$x = 5.63$ (G2) 4

(e) (i) $f(x) = 1$ when $\cos \hat{OPA} = 1$ (R1)

hence, when $\hat{OPA} = 0$. (R1)

This occurs when the points O, A, P are collinear. (R1)

(ii) The line (OA) has equation $y = \dfrac{3x}{4}$ (M1)

When $y = 10$, $x = \dfrac{40}{3}$ $(= 13\frac{1}{3})$ (A1)

OR

$x = \dfrac{40}{3}$ $(= 13\frac{1}{3})$ (G2) 5

Note: *Award (G1) for 13.3.*

[16]

57. (a) Area $= \dfrac{1}{2} r^2 \theta = \dfrac{1}{2}(15^2)(2)$ (M1)

$= 225$ (cm^2) (A1) (C2)

(b) Area $\Delta OAB = \dfrac{1}{2} 15^2 \sin 2 = 102.3$ (A1)

Area $= 225 - 102.3 = 122.7$ (cm^2)

$= 123$ (3 sf) (A1) (C2)

[4]

58. (a) $\dfrac{\sin(\hat{ACB})}{20} = \dfrac{\sin 50°}{17}$ (M1)

$\Rightarrow \sin(\hat{ACB}) = \dfrac{20 \sin 50°}{17} = 0.901$

$\hat{ACB} > 90° \Rightarrow \hat{ACB} = 180° - 64.3° = 115.7°$

$\hat{ACB} = 116$ (3 sf) (A1) (C2)

(b) In Triangle 1, $\hat{ACB} = 64.3°$

$\Rightarrow \hat{BAC} = 180° - (64.3° + 50°)$

$= 65.7°$ (A1)

Area $= \dfrac{1}{2}(20)(17) \sin 65.7° = 155$ (cm^2) (3 sf) (A1) (C2)

[4]

59. **METHOD 1**

The value of cosine varies between –1 and +1. Therefore:

$t = 0 \Rightarrow a + b = 14.3$

$t = 6 \Rightarrow a - b = 10.3$

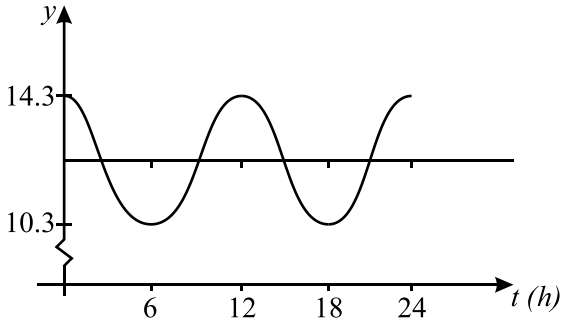
$\Rightarrow 2a = 24.6 \Rightarrow a = 12.3$ (A1) (C1)

$\Rightarrow 2b = 4.0 \Rightarrow b = 2$ (A1) (C1)

Period = 12 hours $\Rightarrow \dfrac{2\pi(12)}{k} = 2\pi$ (M1)

$\Rightarrow k = 12$ (A1) (C2)

METHOD 2



From consideration of graph: Midpoint = $a = 12.3$ (A1) (C1)

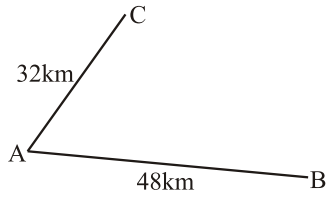
Amplitude = $b = 2$ (A1) (C1)

Period = $\dfrac{2\pi}{\frac{2\pi}{k}} = 12$ (M1)

$\Rightarrow k = 12$ (A1) (C2)

[4]

60.



$\cos C\hat{A}B = \dfrac{48^2 + 32^2 - 56^2}{2(48)(32)}$ (M1)(A1)

$C\hat{A}B = \arccos(0.0625)$ (A1)

$\approx 86°$ (A1)

[4]

61. (a) $2\cos^2 x + \sin x = 2(1 - \sin^2 x) + \sin x$

$= 2 - 2\sin^2 x + \sin x$ (A1)

(b) $2\cos^2 x + \sin x = 2$

$\Rightarrow 2 - 2\sin^2 x + \sin x = 2$

$\sin x - 2\sin^2 x = 0$

$\sin x(1 - 2\sin x) = 0$

$\sin x = 0$ or $\sin x = \dfrac{1}{2}$ (M1)

$\sin x = 0 \Rightarrow x = 0$ or π (0° or 180°) (A1)

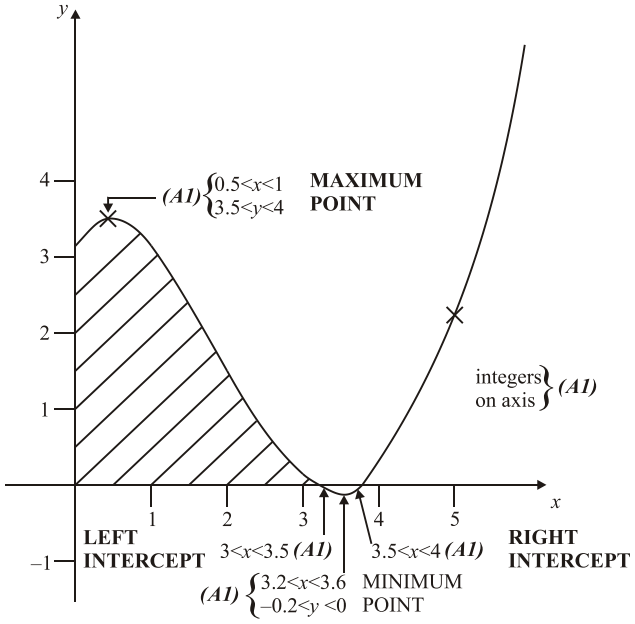
Note: *Award (A1) for both answers.*

$\sin x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ (30° or 150°) (A1)

Note: *Award (A1) for both answers.*

[4]

62. (a)



5

(b) π is a solution if and only if $\pi + \pi\cos\pi = 0$. (M1)

Now $\pi + \pi\cos\pi = \pi + \pi(-1)$ (A1)

$= 0$ (A1) 3

(c) By using appropriate calculator functions $x = 3.696\,722\,9...$ (M1)

$\Rightarrow x = 3.69672$ (6sf) (A1) 2

(d) See graph: (A1)

$\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x$ (A1) 2

(e) **EITHER** $\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x = 7.86960$ (6 sf) (A3) 3

Note: *This answer assumes appropriate use of a calculator eg*

'fnInt': $\begin{cases} fnInt(Y_1, X, 0, \pi) = 7.869604401 \\ with\ Y_1 = \pi + x\cos x \end{cases}$

OR $\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x = [\pi x + x\sin x + \cos x]_0^{\pi}$

$= \pi(\pi - 0) + (\pi\sin\pi - 0 \times \sin 0) + (\cos\pi - \cos 0)$ (A1)

$= \pi^2 + 0 + -2 = 7.86960$ (6 sf) (A1) 3

[15]

63. (a) (i) $Q = \frac{1}{2}(14.6 - 8.2)$ (M1)

$= 3.2$ (A1)

(ii) $P = \frac{1}{2}(14.6 + 8.2)$ (M0)

$= 11.4$ (A1) 3

(b) $10 = 11.4 + 3.2\cos\left(\frac{\pi}{6}t\right)$ (M1)

so $\frac{-7}{16} = \cos\left(\frac{\pi}{6}t\right)$

therefore $\arccos\left(\frac{-7}{16}\right) = \frac{\pi}{6}t$ (A1)

which gives $2.0236... = \frac{\pi}{6}t$ or $t = 3.8648$. $t = 3.86$(3 sf) (A1) 3

(c) (i) By symmetry, next time is $12 - 3.86... = 8.135...$ $t = 8.14$ (3 sf) (A1)

(ii) From above, first interval is $3.86 < t < 8.14$ (A1)

This will happen again, 12 hours later, so (M1)

$15.9 < t < 20.1$ (A1) 4

[10]

64. $3\cos x = 5\sin x$

$\Rightarrow \frac{\sin x}{\cos x} = \frac{3}{5}$ (M1)

$\Rightarrow \tan x = 0.6$ (A1)

$x = 31°$ or $x = 211°$ (to the nearest degree) (A1)(A1)

(C2)(C2)

Note: *Deduct [1 mark] if there are more than two answers.*

[4]

65. $\sin A = \frac{5}{13} \Rightarrow \cos A = \pm\frac{12}{13}$ (A1)

But A is obtuse $\Rightarrow \cos A = -\frac{12}{13}$ (A1)

$\sin 2A = 2\sin A\cos A$ (M1)

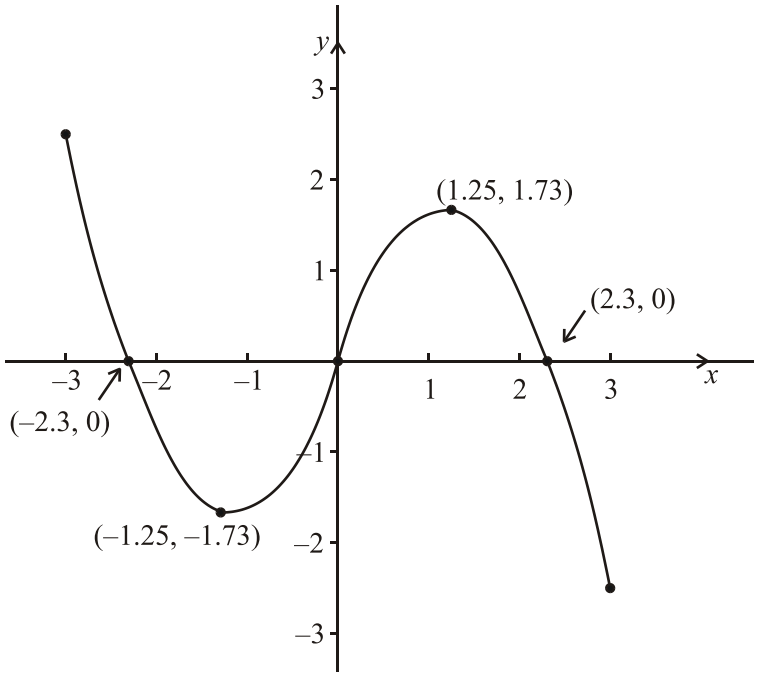
$= 2 \times \frac{5}{13} \times \left(-\frac{12}{13}\right)$

$= -\frac{120}{169}$ (A1)

(C4)

[4]

66. (a) $y = \pi \sin x - x$



(A5) 5

Notes: *Award (A1) for appropriate scales marked on the axes.*
Award (A1) for the x-intercepts at (±2.3, 0).
Award (A1) for the maximum and minimum points at (±1.25, ±1.73).

Award (A1) for the end points at (±3, ±2.55).
Award (A1) for a smooth curve.
Allow some flexibility, especially in the middle three marks here.

(b) $x = 2.31$ (A1) 1

(c) $\int (\pi \sin x - x)\mathrm{d}x = -\pi \cos x - \frac{x^2}{2} + C$ (A1)(A1)

Note: *Do not penalize for the absence of C.*

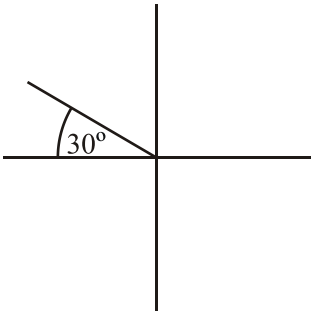
Required area = $\int_0^1 (\pi \sin x - x)\mathrm{d}x$ (M1)

$= 0.944$ (G1)

OR area = 0.944 (G2) 4

[10]

67. (a)



Acute angle 30° (M1)

Note: *Award the (M1) for 30° and/or quadrant diagram/graph seen.*

2nd quadrant since sine positive and cosine negative

$\Rightarrow \theta = 150°$ (A1)

(C2)

(b) $\tan 150° = -\tan 30°$ **or** $\tan 150° = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ (M1)

$\tan 150° = -\frac{1}{\sqrt{3}}$ (A1)

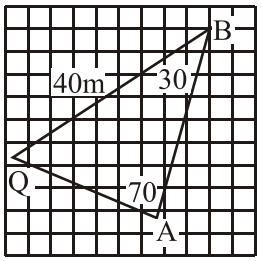
(C2)

[4]

68. (a) $\frac{PQ}{40} = \tan 36°$

$\Rightarrow PQ \approx 29.1$ m (3 sf) (A1) (C1)

(b)



$A\hat{Q}B = 80°$ (A1)

$\frac{AB}{\sin 80°} = \frac{40}{\sin 70°}$ (M1)

Note: *Award (M1) for correctly substituting.*

$\Rightarrow$ AB = 41 9. m (3 sf) (A1)

(C3)

[4]

69. Perimeter = $5(2\pi - 1) + 10$ (M1)(A1)(A1)

Note: *Award (M1) for working in radians; (A1) for $2\pi - 1$; (A1) for $+10$.*

$= (10\pi + 5)$ cm (= 36.4, to 3 sf) (A1)

(C4)

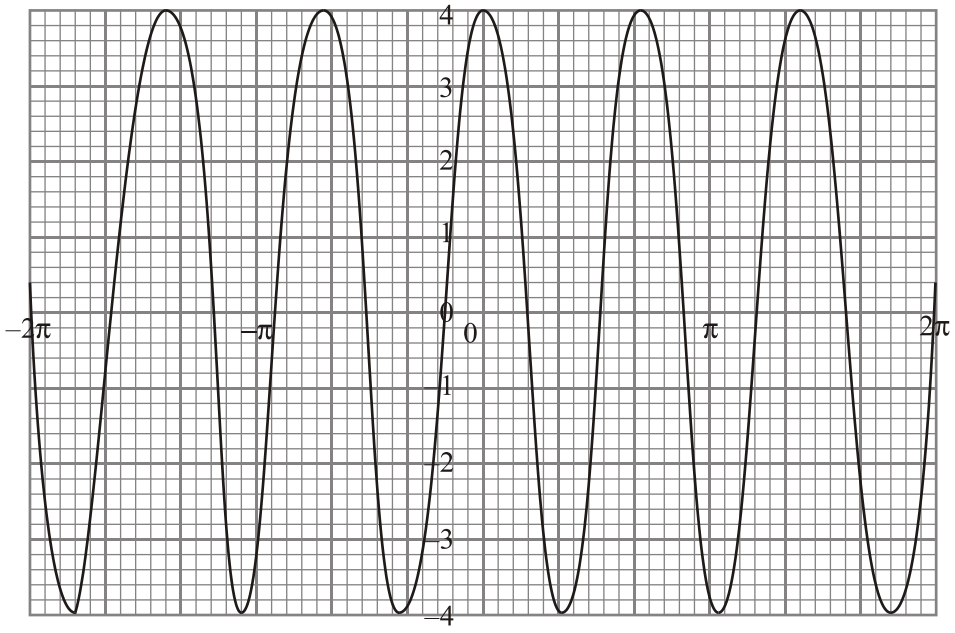
[4]

70. From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$ (M2)

or by observing $|\sin\theta| \leq 1$.

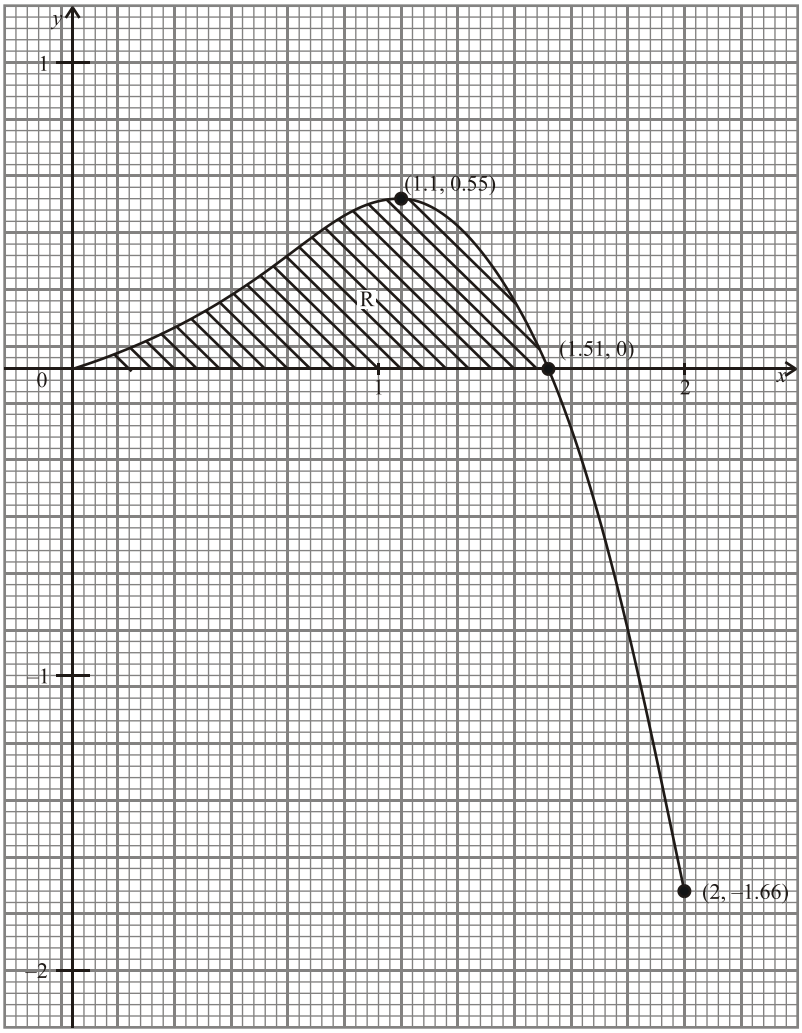
$k > 4$, $k < -4$ (A1)(A1)

(C2)(C2)



[4]

71. (a)(i) & (c)(i)



(A3)

Notes: *The sketch does **not** need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and (2, –1.66) should be indicated in some way.*
Award (A1) for the correct shape.
Award (A2) for 3 or 4 correctly indicated points, (A1) for 1 or 2 points.

(ii) Approximate positions are
positive x-intercept (1.57, 0) (A1)
maximum point (1.1, 0.55) (A1)
end points (0, 0) and (2, –1.66) (A1)(A1) 7

(b) $x^2 \cos x = 0 \qquad x \neq 0 \Rightarrow \cos x = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (A1) 2

Note: *Award (A2) if answer correct.*

(c) (i) see graph (A1)

(ii) $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ (A2) 3

Note: *Award (A1) for limits, (A1) for rest of integral correct (do not penalize missing dx).*

(d) Integral = 0.467 (G3)
OR

Integral = $\left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\pi/2}$ (M1)

$= \left[\frac{\pi^2}{4}(1) + 2\left(\frac{\pi}{2}\right)(0) - 2(1)\right] - [0 + 0 - 0]$ (M1)

$= \frac{\pi}{2} - 2$ (exact) **or** 0.467 (3 sf) (A1) 3

[15]

72. (a) From graph, period = 2π (A1) 1

(b) Range = $\{y \mid -0.4 < y < 0.4\}$ (A1) 1

(c) (i) $f'(x) = \frac{d}{dx}\{\cos x\,(\sin x)^2\}$

$= \cos x\,(2\sin x\cos x) - \sin x\,(\sin x)^2$ **or** $-3\sin^3 x + 2\sin x$ (M1)(A1)(A1)

Note: *Award (M1) for using the product rule and (A1) for each part.*

(ii) $f'(x) = 0$ (M1)

$\Rightarrow \sin x\{2\cos x - \sin^2 x\} = 0$ **or** $\sin x\{3\cos x - 1\} = 0$ (A1)

$\Rightarrow 3\cos^2 x - 1 = 0$

$\Rightarrow \cos x = \pm\sqrt{\left(\frac{1}{3}\right)}$ (A1)

At A, $f(x) > 0$, hence $\cos x = \sqrt{\left(\frac{1}{3}\right)}$ (R1)(AG)

(iii) $f(x) = \sqrt{\left(\frac{1}{3}\right)\left(1 - \left(\sqrt{\left(\frac{1}{3}\right)}\right)^2\right)}$ (M1)

$= \frac{2}{3}\times\frac{1}{\sqrt{3}} = \frac{2}{9}\sqrt{3}$ (A1) 9

(d) $x = \frac{\pi}{2}$ (A1) 1

(e) (i) $\int (\cos x)(\sin x)^2\,dx = \frac{1}{3}\sin^3 x + c$ (M1)(A1)

(ii) Area = $\int_0^{\pi/2} (\cos x)(\sin x)^2\,dx = \frac{1}{3}\left\{\left(\sin\frac{\pi}{2}\right)^3 - (\sin 0)^3\right\}$ (M1)

$= \frac{1}{3}$ (A1) 4

(f) At C $f''(x) = 0$ (M1)

$\Leftrightarrow 9\cos^3 x - 7\cos x = 0$

$\Leftrightarrow \cos x(9\cos^2 x - 7) = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (reject) ***or*** $x = \arccos\frac{\sqrt{7}}{3} = 0.491$ (3 sf) (A1)(A1) 4

[20]

73. ***Note:*** *Award (M1) for identifying the largest angle.*

$\cos\alpha = \frac{4^2 + 5^2 - 7^2}{2\times 4\times 5}$ (M1)

$= -\frac{1}{5}$ (A1)

$\Rightarrow \alpha = 101.5°$ (A1)

OR Find other angles first

$\beta = 44.4°$ $\gamma = 34.0°$ (M1)

$\Rightarrow \alpha = 101.6°$ (A1)(A1)

(C4)

Note: *Award (C3) if not given to the correct accuracy.*

[4]

74. $AB = r\theta$

$= \frac{1}{2}r^2\theta \times \frac{2}{r}$ (M1)(A1)

$= 21.6 \times \frac{2}{5.4}$ (A1)

$= 8$ cm (A1)

OR $\frac{1}{2} \times (5.4)^2\theta = 21.6$

$\Rightarrow \theta = \frac{4}{2.7}$ (= 1.481 radians) (M1)

$AB = r\theta$ (A1)

$= 5.4 \times \frac{4}{2.7}$ (M1)

$= 8$ cm (A1)

(C4)

[4]

75. (a) $\left|\overrightarrow{OA}\right| = 6 \Rightarrow A$ is on the circle (A1)

$\left|\overrightarrow{OB}\right| = 6 \Rightarrow B$ is on the circle. (A1)

$\left|\overrightarrow{OC}\right| = \left|\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}\right|$

$= \sqrt{25+11}$

$= 6 \Rightarrow C$ is on the circle. (A1) 3

(b) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ (M1)

$= \begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix}$ (A1) 2

(c) $\cos O\hat{A}C = \frac{\overrightarrow{AO} \cdot \overrightarrow{AC}}{|AO||AC|}$ (M1)

$= \frac{\begin{pmatrix} -6 \\ 0 \end{pmatrix}\begin{pmatrix} -1 \\ \sqrt{11} \end{pmatrix}}{6\sqrt{1+11}}$

$= \frac{6}{6\sqrt{12}}$ (A1)

$= \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$ (A1)

OR $\cos O\hat{A}C = \dfrac{6^2 + (\sqrt{12})^2 - 6^2}{2 \times 6 \times \sqrt{12}}$ (M1)(A1)

$= \dfrac{1}{\sqrt{12}}$ as before (A1)

OR using the triangle formed by $\overrightarrow{AC}$ and its horizontal and vertical components:

$\left|\overrightarrow{AC}\right| = \sqrt{12}$ (A1)

$\cos O\hat{A}C = \dfrac{1}{\sqrt{12}}$ (M1)(A1) 3

Note: *The answer is 0.289 to 3 sf*

(d) A number of possible methods here

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix} - \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ (A1)

$= \begin{pmatrix} 11 \\ \sqrt{11} \end{pmatrix}$ (A1)

$|BC| = \sqrt{132}$

$|\Delta ABC| = \dfrac{1}{2} \times \sqrt{132} \times \sqrt{12}$ (A1)

$= 6\sqrt{11}$ (A1)

OR ΔABC has base $AB = 12$ (A1)

and height $= \sqrt{11}$ (A1)

$\Rightarrow$ area $= \dfrac{1}{2} \times 12 \times \sqrt{11}$ (A1)

$= 6\sqrt{11}$ (A1)

OR Given $\cos B\hat{A}C = \dfrac{\sqrt{3}}{6}$

$\sin B\hat{A}C = \dfrac{\sqrt{33}}{6} \Rightarrow |\Delta ABC| = \dfrac{1}{2} \times 12 \times \sqrt{12} \times \dfrac{\sqrt{33}}{6}$ (A1)(A1)(A1)

$= 6\sqrt{11}$ (A1) 4

[12]

76. $\tan^2 x = \dfrac{1}{3}$ (M1)

$\Rightarrow \tan x = \pm \dfrac{1}{\sqrt{3}}$ (M1)

$\Rightarrow x = 30°$ or $x = 150°$ (A1)(A1) (C2)(C2)

[4]

77. $h = r$ so $2r^2 = 100 \Rightarrow r^2 = 50$ (M1)

$l = 10\theta = 2\pi r$ (M1)

$\Rightarrow \theta = \dfrac{2\pi\sqrt{50}}{10}$ (A1)

$= \frac{2\pi 5\sqrt{2}}{10}$

$\theta = \pi\sqrt{2} = 4.44$ (3sf) (A1)

(C4)

Note: *Accept either answer.*

[4]

78. (a) $f(1) = 3$ $f(5) = 3$ (A1)(A1)2

(b) **EITHER** distance between successive maxima = period (M1)

$= 5 - 1$ (A1)

$= 4$ (AG)

OR Period of $\sin kx = \frac{2\pi}{k}$; (M1)

so period $= \frac{2\pi}{\frac{\pi}{2}}$ (A1)

$= 4$ (AG) 2

(c) **EITHER** $A \sin\left(\frac{\pi}{2}\right) + B = 3$ and $A \sin\left(\frac{3\pi}{2}\right) + B = -1$ (M1) (M1)

$\Leftrightarrow A + B = 3, -A + B = -1$ (A1)(A1)

$\Leftrightarrow A = 2, B = 1$ (AG)(A1)

OR Amplitude = A (M1)

$A = \frac{3-(-1)}{2} = \frac{4}{2}$ (M1)

$A = 2$ (AG)

Midpoint value = B (M1)

$B = \frac{3+(-1)}{2} = \frac{2}{2}$ (M1)

$B = 1$ (A1) 5

Note: *As the values of A = 2 and B = 1 are likely to be quite obvious to a bright student, do not insist on too detailed a proof.*

(d) $f(x) = 2 \sin\left(\frac{\pi}{2}x\right) + 1$

$f'(x) = \left(\frac{\pi}{2}\right) 2\cos\left(\frac{\pi}{2}x\right) + 0$ (M1)(A2)

Note: *Award (M1) for the chain rule, (A1) for* $\left(\frac{\pi}{2}\right)$*, (A1) for* $2\cos\left(\frac{\pi}{2}x\right)$.

$= \pi \cos\left(\frac{\pi}{2}x\right)$ (A1) 4

Notes: *Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.*

(e) (i) $y = k - \pi x$ is a tangent $\Rightarrow -\pi = \pi \cos\left(\frac{\pi}{2}x\right)$ (M1)

$\Rightarrow -1 = \cos\left(\frac{\pi}{2}x\right)$ (A1)

$\Rightarrow \frac{\pi}{2}x = \pi$ or 3π or ...

$\Rightarrow x = 2$ or 6 ... (A1)

Since $0 \le x \le 5$, we take $x = 2$, so the point is (2, 1) (A1)

(ii) Tangent line is: $y = -\pi(x - 2) + 1$ (M1)

$y = (2\pi + 1) - \pi x$

$k = 2\pi + 1$ (A1) 6

(f) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2$ (A1)

$\Rightarrow \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$ (A1)

$\Rightarrow \frac{\pi}{2}x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$

$x = \frac{1}{3}$ or $\frac{5}{3}$ or $\frac{13}{3}$ (A1)(A1)(A1) 5

[24]