

SL Matrix Practice problems

1. The matrices A, B, X are given by

$$A = \begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}, B = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}, X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ where } a, b, c, d \in \mathbb{Q}.$$

Given that $AX + X = B$, find the **exact** values of a, b, c and d .

(Total 8 marks)

2. Consider the matrix $A = \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix}$.

(a) Write down the inverse, A^{-1} .

(2)

(b) B, C and X are also 2×2 matrices.

(i) Given that $XA + B = C$, express X in terms of A^{-1}, B and C .

(ii) Given that $B = \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix}$, and $C = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$, find X .

(4)

(Total 6 marks)

3. Let $A = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$. Giving your answers in terms of a, b, c, d and e ,

(a) write down $A + B$;

(b) find AB .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(Total 6 marks)

4. (a) Write down the inverse of the matrix $A = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$

(b) **Hence** solve the simultaneous equations

$$x - 3y + z = 1$$

$$2x + 2y - z = 2$$

$$x - 5y + 3z = 3$$

.....

(Total 6 marks)

5. Let $C = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$.

The 2×2 matrix Q is such that $3Q = 2C - D$

(a) Find Q . **(3)**

(b) Find CD . **(4)**

(c) Find D^{-1} . **(2)**

(Total 9 marks)

6. Matrices A , B and C are defined by

$$A = \begin{pmatrix} 5 & 1 \\ 7 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 4 \\ -3 & 15 \end{pmatrix} \quad C = \begin{pmatrix} 9 & -7 \\ 8 & 2 \end{pmatrix}.$$

Let X be an unknown 2×2 matrix satisfying the equation

$$AX + B = C.$$

This equation may be solved for X by rewriting it in the form $X = A^{-1}D$, where D is a 2×2 matrix.

(a) Write down A^{-1} . **(2)**

(b) Find D . **(3)**

(c) Find X . **(2)**

(Total 7 marks)

7. (a) Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

(i) Write down the value of a .

(ii) Find the value of b .

(b) Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ 9 & 23 \end{pmatrix}$.

Find the value of q .

.....

.....

.....

.....

.....

.....

.....

.....

(Total 6 marks)

8. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) Write down the inverse matrix A^{-1} .

(b) Consider the equation $AX = B$.

(i) Express X in terms of A^{-1} and B .

(ii) **Hence**, solve for X .

.....

.....

.....

.....

(Total 6 marks)

10. Let $A = \begin{pmatrix} 3 & 2 \\ k & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find, in terms of k ,

- (a) $2A - B$;
- (b) $\det(2A - B)$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(Total 6 marks)

11. The function f is given by $f(x) = mx^3 + nx^2 + px + q$, where m, n, p, q are integers. The graph of f passes through the point $(0, 0)$.

- (a) Write down the value of q . (1)

The graph of f also passes through the point $(3, 18)$.

- (b) Show that $27m + 9n + 3p = 18$.

The graph of f also passes through the points $(1, 0)$ and $(-1, -10)$. (2)

- (c) Write down the other two linear equations in m, n and p . (2)

- (d) (i) Write down these three equations as a matrix equation.
- (ii) Solve this matrix equation. (6)

- (e) The function f can also be written $f(x) = x(x-1)(rx-s)$ where r and s are integers. Find r and s .

(3)
(Total 14 marks)