SL Vector Practice Set 2

1. The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$.

(8)

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1.

(2)

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point C $(k, -k, -5)$ is on L_2. Find the coordinates of C.

(6)

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C.

(2)

(Total 18 marks)

2. In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point (3, 2, 7). Its position vector after t minutes is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}.$$

(a) Find the speed of the model airplane.

(2)

At 13:00 the second model airplane is at the point (– 5, 10, 23). After two minutes, it is at the point (3, 16, 39).

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$.

(3)

(c) The airplanes meet at point Q.

(i) At what time do the airplanes meet?

(ii) Find the position of Q.

(6)

(d) Find the angle θ between the paths of the two airplanes.

(6)

(Total 17 marks)

3. A triangle has its vertices at A(–1, 3), B(3, 6) and C(–4, 4).

(a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -9$

(b) Show that, to three significant figures, $\cos B\hat{A}C = -0.569$.

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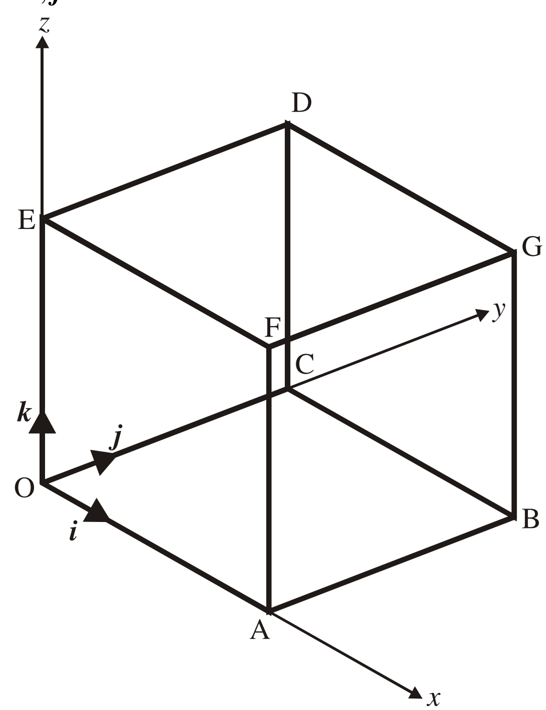
(Total 6 marks)

4. The diagram shows a cube, OABCDEFG where the length of each edge is 5cm. Express the following vectors in terms of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$.



(a) $\overrightarrow{OG}$;

(b) $\overrightarrow{BD}$;

(c) $\overrightarrow{EB}$.

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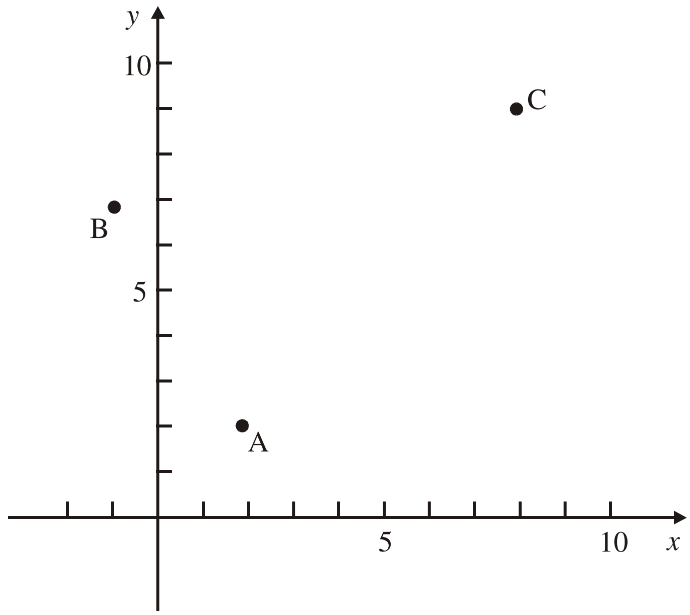
(Total 6 marks)

5. The diagram shows points A, B and C which are three vertices of a parallelogram ABCD. The point A has position vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.



(a) Write down the position vector of B and of C.

(2)

(b) The position vector of point D is $\begin{pmatrix} d \\ 4 \end{pmatrix}$. Find d.

(3)

(c) Find $\overrightarrow{BD}$.

(1)

The line L passes through B and D.

(d) (i) Write down a vector equation of L in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} m \\ n \end{pmatrix}.$$

(ii) Find the value of t at point B.

(3)

(e) Let P be the point (7, 5). By finding the value of t at P, show that P lies on the line L.

(3)

(f) Show that $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{BD}$.

(4)

(Total 16 marks)

SL Vector Practice Set 2 MarkScheme

1. (a) (i) evidence of approach — M1

eg $\vec{AO} + \vec{OB} = \vec{AB}$, B − A

$\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$ — AG N0

(ii) for choosing **correct** vectors, ($\vec{AO}$ with $\vec{AB}$, or $\vec{OA}$ with $\vec{BA}$) — (A1)(A1)

Note: *Using* $\vec{AO}$ *with* $\vec{BA}$ *will lead to* $\pi - 0.799$. *If they then say* $B\hat{A}O = 0.799$, *this is a correct solution.*

calculating $\vec{AO} \bullet \vec{AB}$, $\left|\vec{AO}\right|$, $\left|\vec{AB}\right|$ — (A1)(A1)(A1)

eg $\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = (-1)(-4) + (2)(6) + (-3)(-1)\ (= 19)$

$\left|d_1\right| = \sqrt{(-1)^2 + 2^2 + (-3)^2}\ \left(= \sqrt{14}\right)$,

$\left|d_2\right| = \sqrt{(-4)^2 + 6^2 + (-1)^2}\ \left(= \sqrt{53}\right)$

evidence of using the formula to find the angle — M1

eg $\cos\theta = \dfrac{(-1)(-4) + (2)(6) + (-3)(-1)}{\sqrt{(-1)^2 + 2^2 + (-3)^2}\sqrt{(-4)^2 + 6^2 + (-1)^2}}$,

$\dfrac{19}{\sqrt{14}\sqrt{53}}$, 0.69751...

$B\hat{A}O = 0.799$ radians (accept 45.8°) — A1 N3

(b) two correct answers — A1A1

eg (1, − 2, 3), (−3, 4, 2), (−7, 10, 1), (−11, 16, 0) — N2

(c) (i) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ — A2 N2

(ii) C on L_2, so $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ — (M1)

evidence of equating components — (A1)

eg $1 - 3t = k$, $-2 + 4t = -k$, $5 = 3 + 2t$

one correct value $t = 1$, $k = -2$ (seen anywhere) — (A1)

coordinates of C are (−2, 2, 5) — A1 N3

(d) for setting up one (or more) correct equation using

$\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ — (M1)

eg $3 + p = -2$, $-8 - 2p = 2$, $-p = 5$

$p = -5$ — A1 N2

[18]

2. (a) speed $= \sqrt{3^2 + 4^2 + 10^2}$ (M1)

$= \sqrt{125} = 5\sqrt{5}$, 11.2, (metres per minute) A1 N2

(b) Let the velocity vector be $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Finding a velocity vector A2

$eg \begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 2\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} 3 \\ 16 \\ 39 \end{pmatrix} - \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix}$

Dividing by 2 to give $\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ A1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ AG N0

(c) (i) At Q, $\begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (M1)

Setting up one correct equation A1

eg $3 + 3t = -5 + 4t$, $2 + 4t = 10 + 3t$, $7 + 10t = 23 + 8t$

$t = 8$ (A1)

Correct answer A1

eg after 8 minutes, 13:08 N3

(ii) Substituting for t (M1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + 8\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$, or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + 8\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

$x = 27$, $y = 34$, $z = 87$ or (27, 34, 87), or $\begin{pmatrix} 27 \\ 34 \\ 87 \end{pmatrix}$ A1 N2

(d) For choosing **both** direction vectors $\boldsymbol{d}_1 = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$ and $\boldsymbol{d}_2 = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$ (A1)

$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 104$, $|\mathrm{d}_1| = \sqrt{125}$, $|\mathrm{d}_2| = \sqrt{89}$ (A1)(A1)(A1)

$\cos\theta = \frac{104}{\sqrt{125}\sqrt{89}} = 0.98601...$ A1

$\theta = 0.167$ (radians) (accept $\theta = 9.59°$) A1 N3

[17]

3. (a) Finding correct vectors, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ A1A1

Substituting correctly in the scalar product

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 4(-3) + 3(1)$ A1

$= -9$ AG 3

(b) $|\overrightarrow{AB}| = 5$ $|\overrightarrow{AC}| = \sqrt{10}$ (A1)(A1)

Attempting to use scalar product formula cos BAC = $\dfrac{-9}{5\sqrt{10}}$ M1

$= -0.569$ (3 s.f) AG 3

[6]

4. (a) $\overrightarrow{OG} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

(b) $\overrightarrow{BD} = 5\boldsymbol{i} + 5\boldsymbol{k}$ A2 2

(c) $\overrightarrow{EB} = 5\boldsymbol{i} + 5\boldsymbol{j} - 5\boldsymbol{k}$ A2 2

Note*: Award A0(A2)(A2) if the 5 is consistently omitted.*

[6]

5. (a) $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$ (A1)(A1) 2

(b) $\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (M1)

$= \begin{pmatrix} 8 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$ (A1)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \left(\text{or } \begin{pmatrix} 8 \\ 9 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$

$d = 11 \left(\text{accept} \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$ (A1) 3

(c) $\overrightarrow{BD} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ (A1) 1

(d) (i) $l : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix} \left(\text{or} \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix} \right)$ (A2)

(ii) At B, $t = 0$ by observation (A1)

OR

$\begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix}$

$\Rightarrow t = 0$ (A1) 3

(e) $\begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix} \Rightarrow 7 + 1 = 12t = 8$

$\Rightarrow t = \dfrac{2}{3}$ (A1)

Note: *The equation* $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ *leads to t = 2.*

when $t = \dfrac{2}{3}$, $y = 7 + \left(\dfrac{2}{3}\right)(-3)$ (M1)

$= 7 - 2 = 5$ (A1)

ie P on line (AG)

OR

$5 - 7 = -3t = -2$

$\Rightarrow t = \dfrac{2}{3}$ (A1)

when $t = \dfrac{2}{3}$, $x = -1 + \dfrac{2}{3} \times 12$ (M1)

$= -1 + 8 = 7$ (A1)

ie P on line (AG) 3

(f) $\overrightarrow{CP} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$ (A1)

$\begin{pmatrix} -1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ -3 \end{pmatrix} = -12 + 12 = 0$ (M1)(A1)

Scalar product of non-zero vectors = 0 $\Rightarrow$ are perpendicular (R1)(AG)

OR

Geometric approach

CP: $m = 4$ (A1)

BD: $m_1 = \dfrac{-1}{4}$ (A1)

$mm_1 = 4 \times \left(\dfrac{-1}{4}\right) = -1$ (A1)

Product of gradients is $-1 \Rightarrow$ lines (vectors) are perpendicular (R1)(AG) 4

[16]