Descriptive Statistics – Practice Problems (99-04) Markscheme

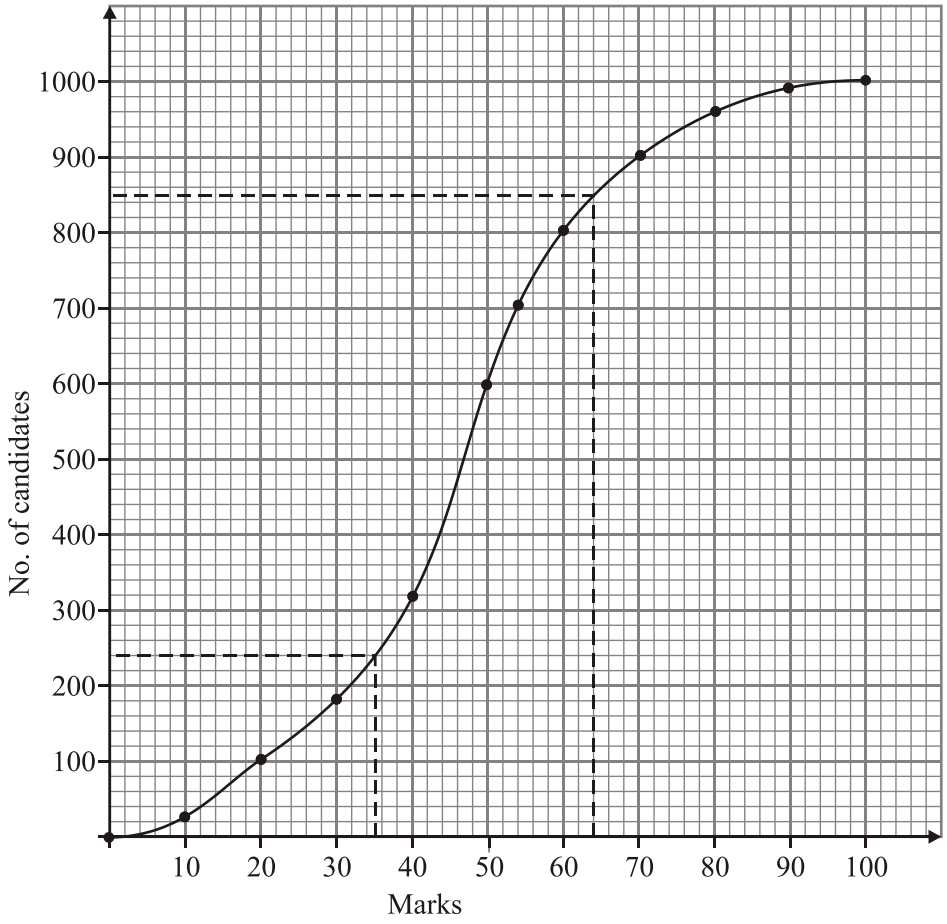
1. (a)

Mark	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80	≤ 90	≤ 100
No. of Candidates	15	65	**165**	335	595	815	905	950	980	**1000**

(A3) 3

Note: *Award (A1) for 165, (A1) for 1000, (A1) if all other entries are correct.*

(b)



(A5) 5

Notes: *Vertical axis and scale* *(A1)*
Horizontal axis and scale *(A1)*
Points *(A1)*
Curve (allow polygon) *(A2)*

(c) (i) Median = 46 (M1)(A1)
(ii) Scores < 35: 240 candidates (M1)(A2)
(iii) Top 15% ⇒ Mark ≥ 63 (M1)(A1)(A1) 8

Notes: *Accept the answers from the* ***student's*** *graph.*
In each part, award (M1) for the dotted lines on the graph.

[16]

2. Mean = $\dfrac{(72\times1.79)+(28\times1.62)}{100}$ (M1)(M1)(M1)

= 1.7424 (= 1.74 to 3 sf) (A1) (C4)

[4]

3. (a) $m = \frac{300}{25}$ (M1)

$= 12$ (A1) (C2)

(b) $s = \sqrt{\left(\frac{625}{25}\right)}$ (M1)

$= 5$ (A1)

(C2)

[4]

4. (a) $\bar{x} = \$59$ (G2)

OR

$\bar{x} = \frac{10\times24+30\times16+...+110\times10+130\times4}{24+16+...+10+4}$ (M1)

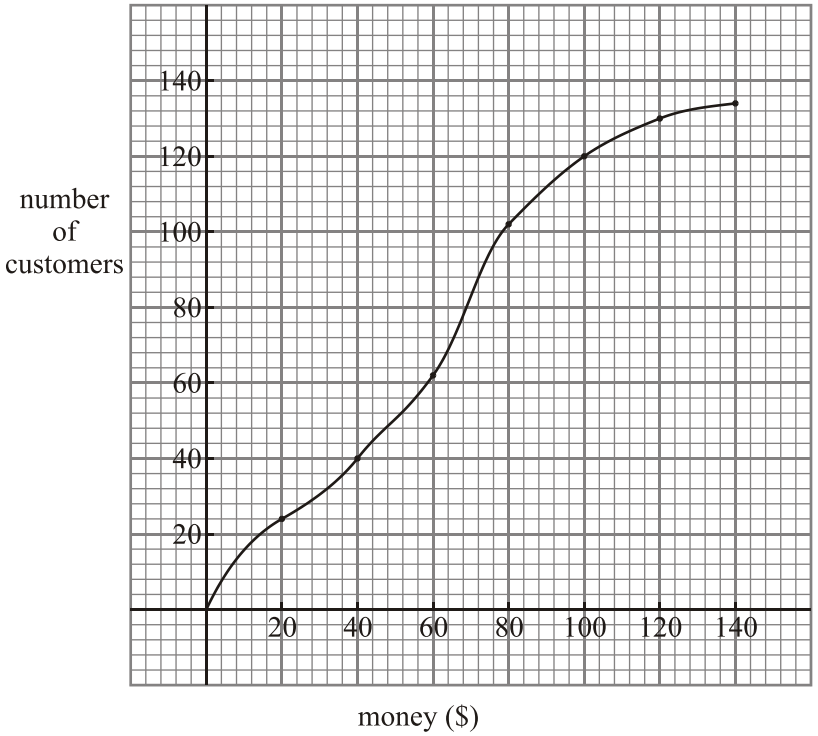
$= \frac{7860}{134}$

$= \$59$ (A1) 2

(b)

Money ($)	<20	<40	<60	<80	<100	<120	<140
Customers	24	40	**62**	**102**	**120**	**130**	**134**

(A1)



(A4) 5

Note: *Award (A1) for the correct scale, (A1) for the points, and (A2) for the curve.*

(c) (i) $t = 2d^{2/3} + 3$

Mean $d = 59$ (M1)

Mean $t \approx 2 \times (59)^{2/3} + 3$ (M1)

≈ 33.3 min. (3 sf) (accept 33.2) (A1)

(ii) $t > 37 \Rightarrow 2d^{2/3} + 3 > 37$ (M1)

$2d^{2/3} > 34$

$d^{2/3} > 17$ (A1)

$d > (17)^{3/2}$

$d > 70.1$

From the graph, when $d = 70.1$, $n = 82$ (A1)

number of shoppers = 134 – **82** (A1)

= **52** (A1) 8

[15]

5. $\dfrac{(10\times1)+(20\times2)+(30\times5)+(40\times k)+(50\times3)}{k+11} = 34$ (M1)(A1)

$\dfrac{40k+350}{k+11} = 34$ (A1)

$\Rightarrow k = 4$ (A1)

(C4)

[4]

6. (a)

x	15	45	75	105	135	165	195	225
f	5	15	33	21	11	7	5	3

(M1)

$\bar{x}$ = 97.2 (exactly) (A1) 2

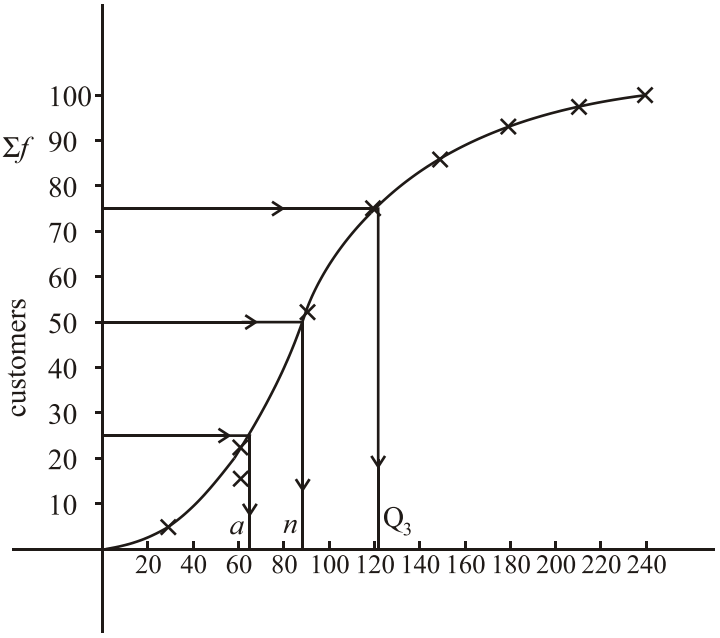
(b)

x	30	60	90	120	150	180	210	240
Σf	5	20	53	74	85	92	97	100

(A1) 1

Note: *Award (A1) for correct values for x, Σf.*

(c)



(A4) 4

Notes: *Award (A2) for 6 or more points correct, (A1) for 4/5 points correct.*
Award (A1) for a reasonable graph, (A1) for the correct axes and the given scales.

(d) Median = 87 ± 2 (A1)
Lower quartile = 65 ± 2 (A1)
Upper quartile = 123 ± 2 (A1) 3

[10]

7. (a) (i) 10 (A1)
(ii) 14 + 10 = 24 (A1) 2

(b)

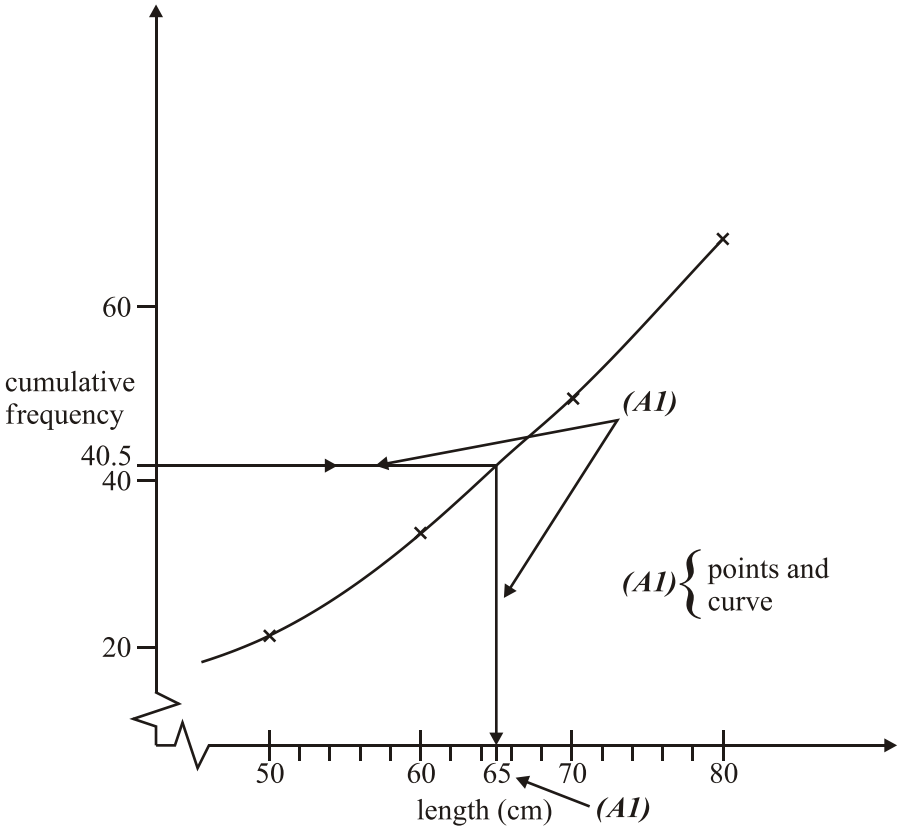
	x_i	f_i	
	15	1	
	25	5	
	35	7	
	45	9	
(A1)	55	10	(A1)
	65	16	
	75	14	
	85	10	
	95	8	
		80	(AG)

Note: *Award (A0) for using the mid-interval values of 14.5, 24.5 etc.*

(i) $\mu = 63$ (A1)
(ii) $\sigma = 20.5$ (3 sf) (A1) 4

(c) Assymetric diagram/distribution (A1) 1

(d)



3

OR Median = 65 (A3) 3

Note: *This answer assumes appropriate use of a calculator with*

correct arguments.

OR Linear interpolation on the table: (M1)

$\left(\frac{48-40.5}{48-32}\right)\times 60+\left(\frac{40.5-32}{48-32}\right)\times 70 = 65$ (2sf) (A1)(A1) 3

[10]

8. (a) Median = middle number of 75 (M1)

= 38th number

= 4 (A1) (C2)

(b) Mean = $\frac{5+18+48+72+100+42}{75}$ (M1)

$= \frac{285}{75}$

= 3.8 (A1)(C2)

[4]

9. (a) s = 7.41(3 sf) (G3) 3

(b)

Weight (W)	$W \le 85$	$W \le 90$	$W \le 95$	$W \le 100$	$W \le 105$	$W \le 110$	$W \le 115$
Number of packets	5	15	**30**	**56**	**69**	**76**	80

(A1)1

(c) (i) From the graph, the median is approximately 96.8.

Answer: 97 (nearest gram). (A2)

(ii) From the graph, the upper or third quartile is approximately 101.2.

Answer: 101 (nearest gram). (A2)4

(d) Sum = 0, since the sum of the deviations from the mean is zero. (A2)

OR

$\sum(W_i-\overline{W}) = \sum W_i - \left(80\frac{\sum W_i}{80}\right) = 0$ (M1)(A1) 2

(e) Let A be the event: $W > 100$, and B the event: $85 < W \le 110$

$P(A|B) = \frac{P(A\cap B)}{P(B)}$ (M1)

$P(A\cap B) = \frac{20}{80}$ (A1)

$P(B) = \frac{71}{80}$ (A1)

$P(A|B) = 0.282$ (A1)

OR

71 packets with weight $85 < W \le 110$. (M1)

Of these, 20 packets have weight $W > 100$. (M1)

Required probability = $\frac{20}{71}$ (A1)

= 0.282 (A1)4

Notes: *Award (A2) for a correct final answer with no reasoning.*
Award up to (M2) for correct reasoning or method.

[14]

10. (a) (Using mid-intervals)

$$\bar{v} = \frac{65(7) + 75(25) + ... + 135(5)}{7 + 25 + ... + 5} \quad \text{(M1)}$$

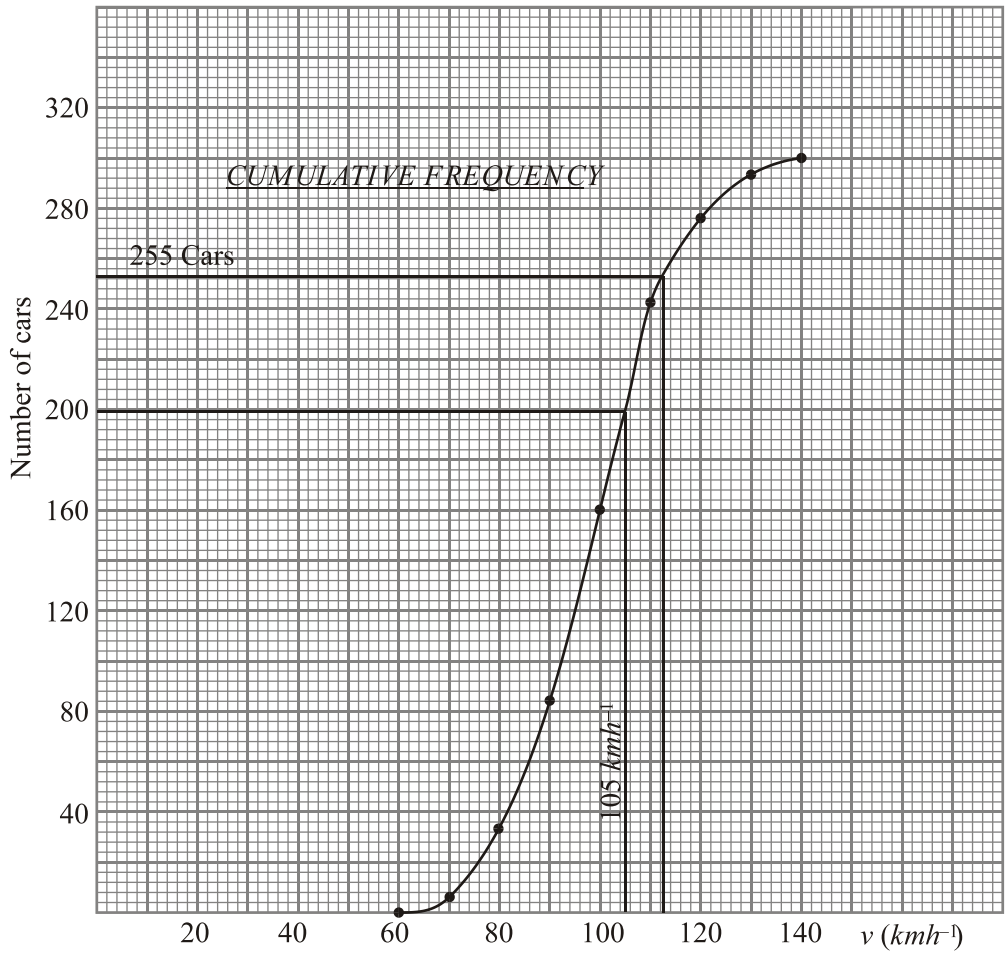
$$= \frac{29450}{300} = 98.2 \text{ km h}^{-1} \quad \text{(A1)}$$

OR

$\bar{v} = 98.2$ (G2)2

(b) (i) $a = 165$, $b = 275$ (A1)

(ii)



(A4)5

Note: *Award (A1) for properly marked scales and axes, (A2) for 9 correctly plotted points, (A1) for 7 or 8 points, (A1) for a smooth curve through the points.*

(c) (i) Vertical line on graph at 105 km h^{-1} (M1)

$$\frac{300 - 200}{300} \times 100\% = 33.3(\pm 1.3\%) \quad \text{(A1)}$$

OR

33.3(±1.3%) (A2)

(ii) 15% of 300 = 45 300 – 45 = 255

Horizontal line on graph at 255 cars (M1)

Speed = 114(± 2 km h^{-1}) (A1)

OR

Speed = 114(± 2 km h^{-1}) (A2)4

[11]

11. Jan–Sept $\sum = 630 \times 9 = 5670$ (M1)(A1)

Oct–Dec $\sum = 810 \times 3 = 2430$ (M1)(A1)

$$\bar{x} = \frac{5670 + 2430}{12}$$ (M1)

mean = 675 (A1) (C6)

[6]

12. (a) (i) median fare = \$24 (±0.5) (A1)

(ii) fare ≤ \$35 => number of cabs is 154 (or 153) (A1) 2

(b) 40% of cabs = 80 cabs (A1)

fares up to \$22 (A1)

distance = \$22 ÷ \$0.55 (M1)

a = 40 km (A1) 4

(c) Distance 90 km => fare = 90 × \$0.55

= \$49.50 (A1)

Fare \$49.50 => number of cabs = 200 –186 (M1)

= 14 (A1)

Thus percentage is $\frac{14}{200}$ = 7% (A1) 4

[10]

13. Median = middle value => $b = 11$ (A1)

Mean = $\frac{a+b+c}{3} = \frac{a+11+c}{3} = 9 \Rightarrow a + 11 + c = 27$ (M1)

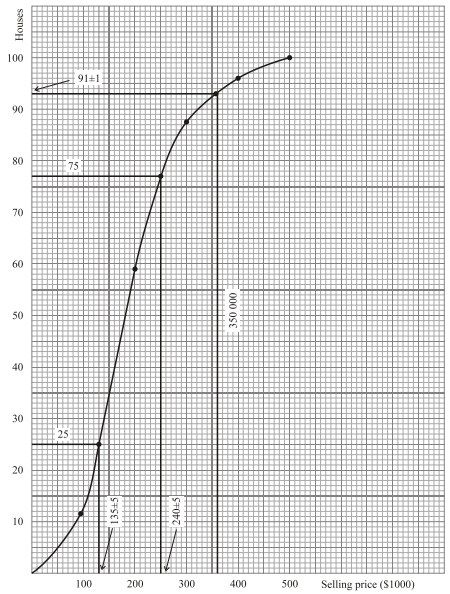
$\Rightarrow a + c = 16$ (A1)

Range = $c - a = 10$ (M1)(A1)

Solving equations simultaneously gives $a = 3$ (A1) (C6)

[6]

14. (a)



(A1)(A2)(A1) 4

Notes: *Award (A1) for correct axes, scales and labelling, (A1) for correctly plotted points.*
Award (A2) for good curve correctly drawn, (A1) for badly drawn, correct curve.

Award (A1) for a correct polygon.

(b) $Q_1 = 135 \pm 5$ $Q_3 = 240 \pm 5$ (M1)(A1)

Interquartile range = 105 ± 10. (Accept 135 – 240 or 240 – 135.) (A1) 3

Note: *Award (M1) for the correct lines on the graph.*

(c) $a = 94 - 87 = 7$, $b = 100 - 94 = 6$ (A1)(A1) 2

(d) mean = $\dfrac{12(50)+46(150)+29(250)+7(350)+6(450)}{100}$ (M1)

= 199 or \$199000 (A1)

OR

mean = 199 or \$199000 (G2) 2

(e) (i) \$350000 => 91.5

Number of *De luxe* houses $\simeq 100 - 91.5$ (M1)

= 9 or 8 (A1)

(ii) P (both > 400000) = $\frac{6}{9}\left(\frac{5}{8}\right)=\frac{5}{12}$ or $\frac{6}{8}\left(\frac{5}{7}\right)=\frac{15}{28}$ (M1)(A1) 4

Note: *Award (M1)(A0) for the answers* $\frac{4}{9}$ or $\frac{9}{16}$ *obtained from correct independent probabilities.*

[15]

15.

x	f	Σf
4	2	2
5	5	7
6	4	11
7	3	14
8	4	18
10	2	20
12	1	21

(a) m = 6 (A2)(C2)

(b) $Q_1 = 5$ (A2)(C2)

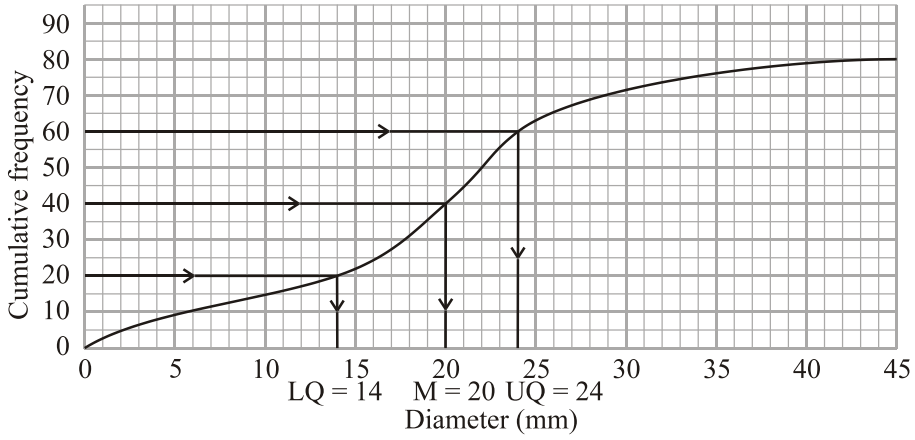
(c) $Q_3 = 8$ (A1)

IQR = 8 – 5 (M1)

= 3 (accept 5 – 8 or [5, 8]) (C2)

[6]

16.



(a) (i) Correct lines drawn on graph, (A1)(C1)
median = 20 (A1)(C1)
(ii) Correct lines drawn on graph, (A1)(C1)
UQ = Q_3 = 24 (A1)(C1)

(b) IQR = $Q_3 - Q_1$(or UQ – LQ) (M1)
= 10 (accept 14 to 24) (A1)
(C2)

Note: *Accept 14 to 24, 24 to 14, 14 – 24 or 24 –14.*

[6]

17. (a)

Mark (x)	$0 \le x < 20$	$20 \le x < 40$	$40 \le x < 60$	$60 \le x < 80$	$80 \le x < 100$
Number of Students	22	**50**(±1)	**66**(±1)	**42**(±1)	20

(A1)(A1)(A1) (C3)

(b) 40th Percentile ⇒ 80th student fails, (mark 42%) (M2)
Pass mark 43% (Accept mark > 42.) (A1)(C3)

[6]

18. List of frequencies with p in the middle

eg 5 + 10, p, 6 + 2 ⇒ 15, 8, or $15 < \frac{23+p}{2}$, or $p > 7$. (M1)

Consideration that $p < 10$ because 2 is the mode or discretionary for further processing. (M1)

Possible values of p are 8 and 9 (A2)(A2)(C6)

[6]

19. (a) line(s) on graph (M1)
median is 183 (A1) (C2)

(b) Lower quartile $Q_1 = 175$ (A1)

Upper quartile $Q_3 = 189$ (A1)

IQR is 14
(Accept 189 – 175, 175 to 189, 189 to 175 and 175 – 189) (M1)(A1)(C4)

[6]

20. $d = 11$; $c = 11$ (A1)(A1)(C1)(C1)

$d - a = 8$ (or $11 - a = 8$) (A1)

$a = 3$ (A1) (C2)

$\frac{3+b+11+11}{4} = 8 \left(\text{or } \frac{\text{sum}}{4} = 8 \right)$ (A1)

$b = 7$ (A1)(C2)

[6]