Final Exam Prep – Practice Problems

1. Consider the arithmetic sequence 2, 5, 8, 11, ...

(a) Find u_{101}. **(3)**

(b) Find the value of n so that $u_n = 152$.

(3)
(Total 6 marks)

2. Consider the infinite geometric sequence 3000, – 1800, 1080, – 648, … .

(a) Find the common ratio.

(2)

(b) Find the 10^{th} term.

(2)

(c) Find the **exact** sum of the infinite sequence.

(2)
(Total 6 marks)

3. Find the term in x^3 in the expansion of $\left(\frac{2}{3}x - 3\right)^8$.

(Total 5 marks)

4. Consider the infinite geometric sequence 3, 3(0.9), $3(0.9)^2$, $3(0.9)^3$, … .

(a) Write down the 10^{th} term of the sequence. Do not simplify your answer.

(1)

(b) Find the sum of the infinite sequence.

(4)
(Total 5 marks)

5. (a) Expand $(x – 2)^4$ and simplify your result.

(3)

(b) Find the term in x^3 in $(3x + 4)(x – 2)^4$.

(3)
(Total 6 marks)

6. Let $f(x) = \ln(x + 5) + \ln 2$, for $x > –5$.

(a) Find $f^{-1}(x)$.

(4)

Let $g(x) = e^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b, \in \mathbb{Z}$.

(3)
(Total 7 marks)

7. Let $f(x) = 3(x + 1)^2 – 12$.

(a) Show that $f(x) = 3x^2 + 6x – 9$.

(2)

(b) For the graph of f

(i) write down the coordinates of the vertex;
(ii) write down the **equation** of the axis of symmetry;
(iii) write down the y-intercept;
(iv) find both x-intercepts.

(8)

(c) **Hence** sketch the graph of f.

(2)

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y-direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. **(3)**

(Total 15 marks)

8. Consider $f(x) = \sqrt{x-5}$.

(a) Find

(i) $f(11)$;

(ii) $f(86)$;

(iii) $f(5)$.

(3)

(b) Find the values of x for which f is undefined.

(2)

(c) Let $g(x) = x^2$. Find $(g \circ f)(x)$.

(2)
(Total 7 marks)

9. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write f in the form $f(x) = 3(x - h)^2 - k$.

(3)

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

(3)
(Total 6 marks)

10. Let $\boldsymbol{M} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$, and $\boldsymbol{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Given that $\boldsymbol{M}^2 - 6\boldsymbol{M} + k\boldsymbol{I} = \boldsymbol{O}$, find k.

(Total 6 marks)

11. Let $f(x) = 2x^2 - 12x + 5$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 - k$.

(3)

(b) Write down the vertex of the graph of f.

(2)

(c) Write down the equation of the axis of symmetry of the graph of f.

(1)

(d) Find the y-intercept of the graph of f.

(2)

(e) The x-intercepts of f can be written as $\frac{p \pm \sqrt{q}}{r}$, where $p, q, r \in \mathbb{Z}$.

Find the value of p, of q, and of r.

(7)
(Total 15 marks)

12. Let $f(x) = \frac{1}{x}$, $x \neq 0$.

(a) Sketch the graph of f.

(2)

The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(b) Find an expression for $g(x)$.

(2)

(c) (i) Find the intercepts of g.

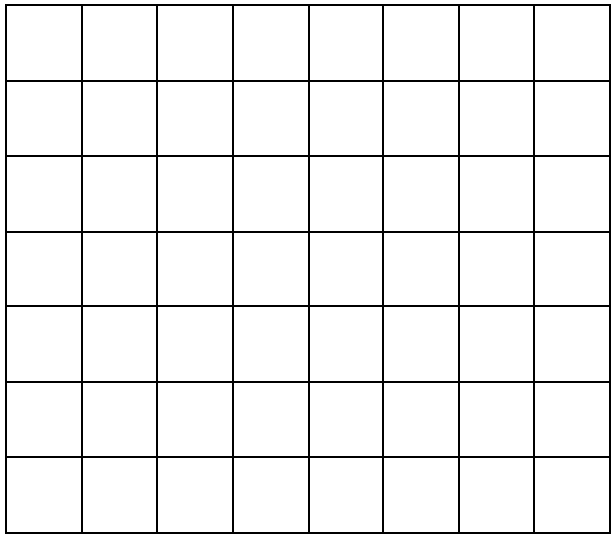
(ii) Write down the equations of the asymptotes of g.

(iii) Sketch the graph of g.

(10)
(Total 14 marks)

13. The function f is defined by $f(x) = \dfrac{3}{\sqrt{9-x^2}}$, for $-3 < x < 3$.n (Calculator OK)

(a) On the grid below, sketch the graph of f.



(2)

(b) Write down the equation of each vertical asymptote.

(2)

(c) Write down the range of the function f.

(2)
(Total 6 marks)

14. The functions f and g are defined by $f : x \mapsto 3x$, $g : x \mapsto x + 2$.

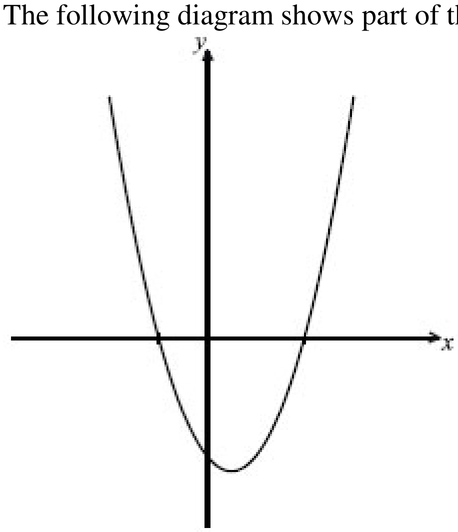
(a) Find an expression for $(f \circ g)(x)$.

(2)

(b) Find $f^{-1}(18) + g^{-1}(18)$.

(4)
(Total 6 marks)

15. The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.



(a) Find both x-intercepts.

(4)

(b) Find the x-coordinate of the vertex.

(2)
(Total 6 marks)

16. In an arithmetic sequence $u_{21} = -37$ and $u_4 = -3$.

(a) Find

(i) the common difference;

(ii) the first term.

(4)

(b) Find S_{10}.

(3)

(Total 7 marks)

17. Let $u_n = 3 - 2n$.

(a) Write down the value of u_1, u_2, and u_3.

(3)

(b) Find $\sum_{n=1}^{20}(3-2n)$.

(3)

(Total 6 marks)

18. Solve the following equations.

(a) $\log_x 49 = 2$

(3)

(b) $\log_2 8 = x$

(2)

(c) $\log_{25} x = -\frac{1}{2}$

(3)

(d) $\log_2 x + \log_2(x - 7) = 3$

(5)

(Total 13 marks)

19. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20th row.

(4)

(b) Calculate the **total** number of seats.

(2)

(Total 6 marks)

20. A sum of \$ 5000 is invested at a compound interest rate of 6.3 % per annum.

(a) Write down an expression for the value of the investment after n full years.

(1)

(b) What will be the value of the investment at the end of five years?

(1)

(c) The value of the investment will exceed \$ 10 000 after n full years.

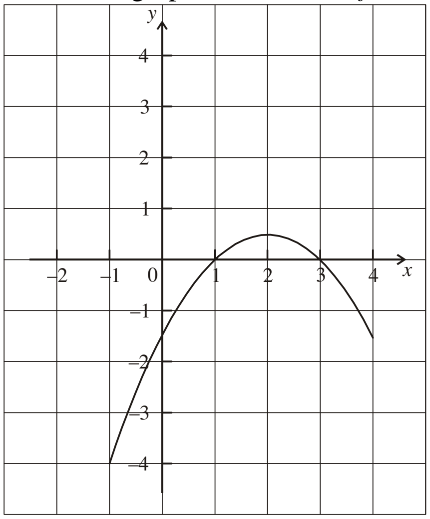
(i) Write down an inequality to represent this information.

(ii) Calculate the minimum value of n.

(4)

(Total 6 marks)

21. Part of the graph of a function f is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$.

(2)

(b) Let $g(x) = f(x + 3)$.

(i) Find $g(-3)$.

(ii) Describe **fully** the transformation that maps the graph of f to the graph of g.

(4)

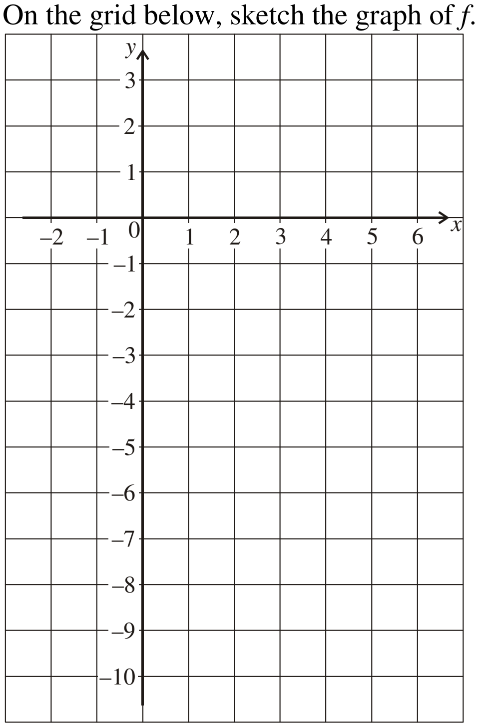
(Total 6 marks)

22. Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f.

(3)

(b) On the grid below, sketch the graph of f.



(3)

(c) Write down the gradient of the graph of f at $x = 2$.

(1)

(Total 7 marks)

23. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n.$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90e^{-0.1n}}.$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

(6)

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

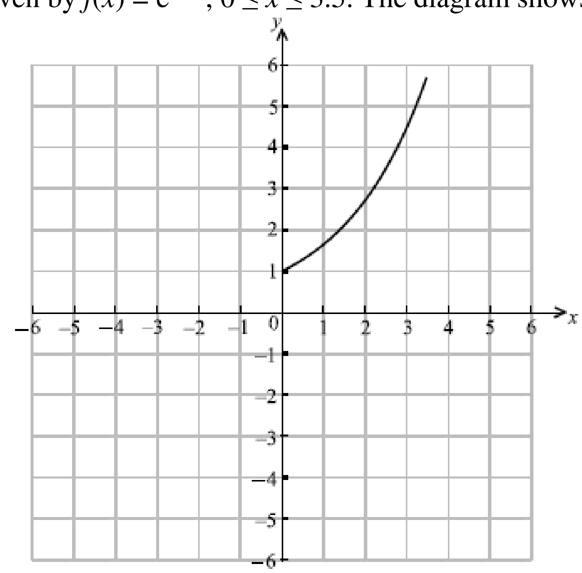
(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)

(Total 17 marks)

24. Let f be the function given by $f(x) = e^{0.5x}$, $0 \le x \le 3.5$. The diagram shows the graph of f.



(a) On the same diagram, sketch the graph of f^{-1}.

(3)

(b) Write down the range of f^{-1}.

(1)

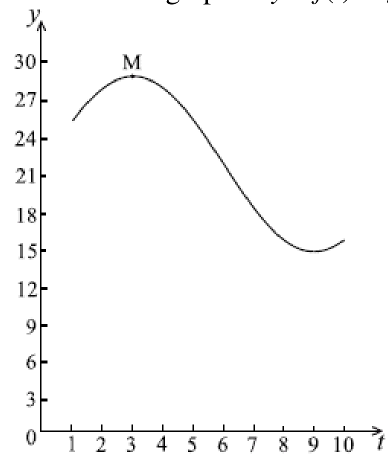
(c) Find $f^{-1}(x)$.

(3)

(Total 7 marks)

25. [***Note: Trig Functions will not be included on the final exam. But transformations of functions will be. Do this problem only if you want to explore transformations more thoroughly.***]

Let $f(t) = a \cos b\,(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \dfrac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

(7)

The transformation P is given by a horizontal stretch of a scale factor of $\dfrac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′.

(2)

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$.

(4)

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f.

(3)
(Total 16 marks)

26. Let $f(x) = 2x^2 + 4x - 6$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$.

(3)

(b) Write down the equation of the axis of symmetry of the graph of f.

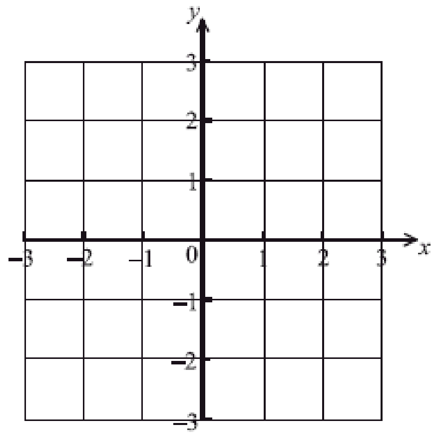
(1)

(c) Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.

(2)
(Total 6 marks)

27. Let $f(x) = x \cos(x - \sin x)$, $0 \leq x \leq 3$.

(a) Sketch the graph of f on the following set of axes.



(3)

(b) The graph of f intersects the x-axis when $x = a$, $a \neq 0$. Write down the value of a.

(1)
(Total 4 marks)

28. Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, k, 2), with (AD) perpendicular to (AB).

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AD}$, giving your answer in terms of k.

(3)

(b) Show that $k = 7$.

(3)

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

(c) Find the position vector of C.

(4)

(d) Find $\cos A\hat{B}C$.

(3)
(Total 13 marks)

29. Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$.
Find the value of p.

(Total 7 marks)

30. The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $B\hat{A}O$.

(8)

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1.

(2)

(c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

(i) Find a vector equation for L_2, giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point C (k, –k, 5) is on L_2. Find the coordinates of C.

(6)

(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C.

(2)
(Total 18 marks)

31. The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

(Total 6 marks)

32. A particle is moving with a constant velocity along line L. Its initial position is A(6, –2, 10). After one second the particle has moved to B(9, –6, 15).

(a) (i) Find the velocity vector, $\overrightarrow{\mathrm{AB}}$.

(ii) Find the speed of the particle.

(4)

(b) Write down an equation of the line L.

(2)
(Total 6 marks)

33. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

Find

(a) $\boldsymbol{A} + \boldsymbol{B}$;

(2)

(b) $-3\boldsymbol{A}$;

(2)

(c) $\boldsymbol{AB}$.

(3)
(Total 7 marks)

34. Let $\boldsymbol{M} = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$.

(a) Write down the determinant of $\boldsymbol{M}$.

(1)

(b) Write down $\boldsymbol{M}^{-1}$.

(2)

(c) **Hence** solve $\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$.

(3)
(Total 6 marks)

35. Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$ in terms of p and q.

(2)

(b) Matrix $\boldsymbol{B}$ is the inverse of matrix $\boldsymbol{A}$. Find the value of p and of q.

(5)

(Total 7 marks)

36. Let $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

The matrix $\boldsymbol{B}$ satisfies the equation $\left(\boldsymbol{I} - \frac{1}{2}\boldsymbol{B}\right)^{-1} = \boldsymbol{A}$, where $\boldsymbol{I}$ is the 3×3 identity matrix.

(b) (i) Show that $\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$.
(ii) Find $\boldsymbol{B}$.
(iii) Write down det $\boldsymbol{B}$.
(iv) **Hence**, explain why $\boldsymbol{B}^{-1}$ exists.

(6)

Let $\boldsymbol{BX} = \boldsymbol{C}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find $\boldsymbol{X}$.
(ii) Write down a system of equations whose solution is represented by $\boldsymbol{X}$.

(5)
(Total 13 marks)

Final Exam Prep – Practice Problems: MarkScheme

1. (a) $d = 3$ (A1)

evidence of substitution into $u_n = a + (n-1)d$ (M1)

e.g. $u_{101} = 2 + 100 \times 3$

$u_{101} = 302$ A1 N3

(b) correct approach (M1)

e.g. $152 = 2 + (n-1) \times 3$

correct simplification (A1)

e.g. $150 = (n-1) \times 3$, $50 = n - 1$, $152 = -1 + 3n$

$n = 51$ A1 N2

[6]

2. (a) evidence of dividing two terms (M1)

e.g. $-\frac{1800}{3000}, -\frac{1800}{1080}$

$r = -0.6$ A1 N2

(b) evidence of substituting into the formula for the 10th term (M1)

e.g. $u_{10} = 3000(-0.6)^9$

$u_{10} = -30.2$ (accept the exact value -30.233088) A1 N2

(c) evidence of substituting into the formula for the infinite sum (M1)

e.g. $S = \frac{3000}{1.6}$

$S = 1875$ A1 N2

[6]

3. evidence of using binomial expansion (M1)

e.g. selecting correct term, $a^8b^0 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + \ldots$

evidence of calculating the factors, in any order A1A1A1

e.g. 56, $\frac{2^3}{3^3}, -3^5, \binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 s.f.) A1 N2

[5]

4. (a) $u_{10} = 3(0.9)^9$ A1 N1

(b) recognizing $r = 0.9$ (A1)

correct substitution A1

e.g. $S = \frac{3}{1-0.9}$

$S = \frac{3}{0.1}$ (A1)

$S = 30$ A1 N3

[5]

5. (a) evidence of expanding M1

e.g. $(x-2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$

$(x-2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$ A2

N2

(b) finding coefficients, $3 \times 24\ (= 72), 4 \times (-8)(= -32)$ (A1)(A1)

term is $40x^3$ A1

N3

[6]

6. (a) **METHOD 1**

$\ln(x+5) + \ln 2 = \ln(2(x+5))\ (= \ln(2x+10))$ (A1)

interchanging x and y (seen anywhere) (M1)

e.g. $x = \ln(2y+10)$

evidence of correct manipulation (A1)

e.g. $e^x = 2y + 10$

$f^{-1}(x) = \frac{e^x - 10}{2}$ A1

N2

METHOD 2

$y = \ln(x+5) + \ln 2$

$y - \ln 2 = \ln(x+5)$ (A1)

evidence of correct manipulation (A1)

e.g. $e^{y - \ln 2} = x + 5$

interchanging x and y (seen anywhere) (M1)

e.g. $e^{x - \ln 2} = y + 5$

$f^{-1}(x) = e^{x - \ln 2} - 5$ A1

N2

(b) **METHOD 1**

evidence of composition in correct order (M1)

e.g. $(g \circ f)(x) = g(\ln(x+5) + \ln 2)$

$= e^{\ln(2(x+5))} = 2(x+5)$

$(g \circ f)(x) = 2x + 10$ A1A1

N2

METHOD 2

evidence of composition in correct order (M1)

e.g. $(g \circ f)(x) = e^{\ln(x+5) + \ln 2}$

$= e^{\ln(x+5)} \times e^{\ln 2} = (x+5)\,2$

$(g \circ f)(x) = 2x + 10$ A1A1

N2

[7]

7. (a) $f(x) = 3(x^2 + 2x + 1) - 12$ A1

$= 3x^2 + 6x + 3 - 12$ A1

$= 3x^2 + 6x - 9$ AG

N0

(b) (i) vertex is $(-1, -12)$ A1A1

N2

(ii) $x = -1$ (**must** be an equation) A1

N1

(iii) $(0, -9)$ A1

N1

(iv) evidence of solving $f(x) = 0$ (M1)

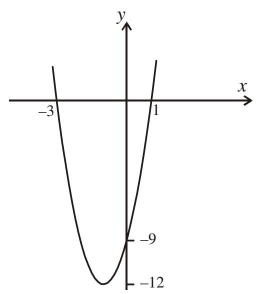
e.g. factorizing, formula,

correct working A1

e.g. $3(x + 3)(x - 1) = 0$, $x = \dfrac{-6 \pm \sqrt{36+108}}{6}$

$(-3, 0)$, $(1, 0)$ A1A1
N1N1

(c)



A1A1
N2

Notes: *Award A1 for a parabola opening upward, A1 for vertex and intercepts in approximately correct positions.*

(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ (accept $p = -1$, $q = -12$, $t = 3$) A1A1A1N3

[15]

8. (a) (i) $\sqrt{6}$ A1 N1

(ii) 9 A1
N1

(iii) 0 A1
N1

(b) $x < 5$ A2
N2

(c) $(g \circ f)(x) = (\sqrt{x-5})^2$ (M1)

$= x - 5$ A1
N2

[7]

9. (a) For a reasonable attempt to complete the square, (or expanding) (M1)

e.g. $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$f(x) = 3(x - 2)^2 - 1$ (accept $h = 2$, $k = 1$) A1A1 N3

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1
N2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1
N2

[6]

10. $\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (A1)

$M^2 = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix}$ A2

$6M = \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix}$ A1

$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ A1

$k = 5$ A1

N2

[6]

11. (a) Evidence of completing the square (M1)

$f(x) = 2(x^2 - 6x + 9) + 5 - 18$ (A1)

$= 2(x - 3)^2 - 13$ (accept $h = 3$, $k = 13$) A1

N3

(b) Vertex is $(3, -13)$ A1A1

N2

(c) $x = 3$ (must be an equation) A1

N1

(d) evidence of using fact that $x = 0$ at y-intercept (M1)

y-intercept is $(0, 5)$ (accept 5) A1

N2

(e) **METHOD 1**

evidence of using $y = 0$ at x-intercept (M1)

e.g. $2(x - 3)^2 - 13 = 0$

evidence of solving this equation (M1)

e.g. $(x - 3)^2 = \frac{13}{2}$ A1

$(x - 3) = \pm\sqrt{\frac{13}{2}}$

$x = 3 \pm \sqrt{\frac{13}{2}} = 3 \pm \frac{\sqrt{26}}{2}$ A1

$x = \frac{6 \pm \sqrt{26}}{2}$

$p = 6$, $q = 26$, $r = 2$ A1A1A1

N4

METHOD 2

evidence of using $y = 0$ at x-intercept (M1)

e.g. $2x^2 - 12x + 5 = 0$

evidence of using the quadratic formula (M1)

$x = \frac{12 \pm \sqrt{12^2 - 4 \times 2 \times 5}}{2 \times 2}$ A1

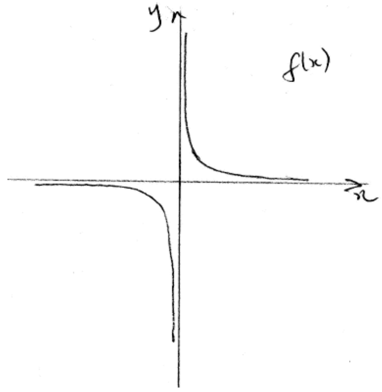
$x = \frac{12 \pm \sqrt{104}}{4}$ $\left(= \frac{6 \pm \sqrt{26}}{2} \right)$ A1

$p = 12$, $q = 104$, $r = 4$ (or $p = 6$, $q = 26$, $r = 2$) A1A1A1

N4

[15]

12. (a)



A1A1 N2

Note: *Award **A1** for the left branch, and **A1** for the right branch.*

(b) $g(x) = \frac{1}{x-2} + 3$ A1A1

N2

(c) (i) Evidence of using $x = 0$ $\left(g(0) = -\frac{1}{2} + 3\right)$ (M1)

$y = \frac{5}{2}$ $(= 2.5)$ A1

evidence of solving $y = 0$ $(1 + 3(x - 2) = 0)$ M1

$1 + 3x - 6 = 0$ (A1)

$3x = 5$

$x = \frac{5}{3}$ A1

Intercepts are $x = \frac{5}{3}$, $y = \frac{5}{2}$ (accept $\left(\frac{5}{3}, 0\right)$ $\left(0, \frac{5}{2}\right)$)

N3

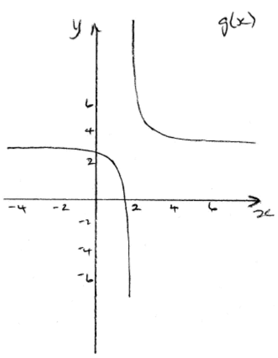
(ii) $x = 2$ A1

N1

$y = 3$ A1

N1

(iii)



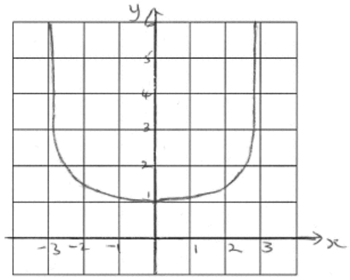
A1A1A1

N3

Note: *Award A1 for the shape (both branches), A1 for the correct behaviour close to the asymptotes, and A1 for the intercepts at approximately* $\left(\frac{5}{3}, 0\right)$ $\left(0, \frac{5}{2}\right)$.

[14]

13. (a)



A1A1 N2

Note: *Award **A1** for the general shape and **A1** for the y-intercept at 1.*

(b) $x = 3, x = -3$ A1A1 N1N1

(c) $y \geq 1$ A2 N2

[6]

14. (a) $(f \circ g): x \mapsto 3(x+2) \quad (= 3x + 6)$ A2 N2

(b) **METHOD 1**

Evidence of finding inverse functions M1

e.g. $f^{-1}(x) = \frac{x}{3} \quad g^{-1}(x) = x - 2$

$f^{-1}(18) = \frac{18}{3} (= 6)$ (A1)

$g^{-1}(18) = 18 - 2 \ (= 16)$ (A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1 N3

METHOD 2

Evidence of solving equations M1

e.g. $3x = 18, x + 2 = 18$

$x = 6, x = 16$ (A1)(A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1 N3

[6]

15. (a) evidence of attempting to solve $f(x) = 0$ (M1)

evidence of correct working A1

e.g. $(x+1)(x-2), \frac{1 \pm \sqrt{9}}{2}$

intercepts are (−1, 0) and (2, 0) (accept $x = -1, x = 2$) A1A1 N1N1

(b) evidence of appropriate method (M1)

e.g. $x_v = \frac{x_1 + x_2}{2}, x_v = -\frac{b}{2a}$, reference to symmetry

$x_v = 0.5$ A1 N2

[6]

16. (a) (i) attempt to set up equations (M1)

$-37 = u_1 + 20d$ and $-3 = u_1 + 3d$ A1

$-34 = 17d$

$d = -2$ A1 N2

	(ii)	$-3 = u_1 - 6 \Rightarrow u_1 = 3$		A1
				N1
	(b)	$u_{10} = 3 + 9 \times -2 = -15$		(A1)
		$S_{10} = \frac{10}{2}(3 + (-15))$		M1
		$= -60$		A1
				N2
				[7]
17.	(a)	$u_1 = 1, u_2 = -1, u_3 = -3$	A1A1A1	N3
	(b)	Evidence of using appropriate formula		M1
		correct values $S_{20} = \frac{20}{2}(2 \times 1 + 19 \times -2)$ $(= 10(2 - 38))$		A1
		$S_{20} = -360$		A1
				N1
				[6]
18.	(a)	$x^2 = 49$	(M1)	
		$x = \pm 7$	(A1)	
		$x = 7$	A1	N3
	(b)	$2^x = 8$		(M1)
		$x = 3$		A1
				N2
	(c)	$x = 25^{-\frac{1}{2}}$		(M1)
		$x = \frac{1}{\sqrt{25}}$		(A1)
		$x = \frac{1}{5}$		A1
				N3
	(d)	$\log_2 (x(x - 7)) = 3$		(M1)
		$\log_2 (x^2 - 7x) = 3$		
		$2^3 = 8 \quad (8 = x^2 - 7x)$		(A1)
		$x^2 - 7x - 8 = 0$		A1
		$(x - 8)(x + 1) = 0$ $(x = 8, x = -1)$		(A1)
		$x = 8$		A1
				N3
				[13]
19.	(a)	Recognizing an AP	(M1)	
		$u_1 = 15 \;\; d = 2 \;\; n = 20$	(A1)	
		substituting into $u_{20} = 15 + (20 - 1) \times 2$	M1	
		$= 53$ (that is, 53 seats in the 20th row)	A1	N2
	(b)	Substituting into $S_{20} = \frac{20}{2}(2(15) + (20 - 1)2)$ (or into $\frac{20}{2}(15 + 53)$)		M1
		$= 680$ (that is, 680 seats in total)		A1
				N2
				[6]
20.	(a)	$5000(1.063)^n$	A1	N1
	(b)	Value = \$ $5000(1.063)^5$ (= \$ 6786.3511...)		
		= \$ 6790 to 3 s.f. (accept \$ 6786, or \$ 6786.35)		A1
				N1

(c) (i) $5000(1.063)^n > 10\,000$ or $(1.063)^n > 2$ A1
N1

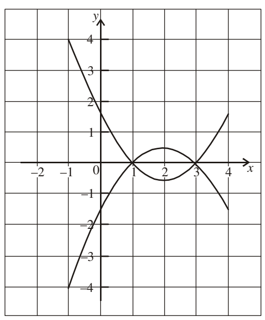
(ii) Attempting to solve the inequality $n\log(1.063) > \log 2$ (M1)
$n > 11.345$ (A1)
12 years A1
N3

Note: *Candidates are likely to use TABLE or LIST on a GDC to find n.*
A good way of communicating this is suggested below.

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ *i.e.* 12 years A1
N3

[6]

21. (a)



M1A1
N2

Note: *Award M1 for evidence of reflection in x-axis, A1 for correct vertex* ***and*** *all intercepts approximately correct.*

(b) (i) $g(-3) = f(0)$ (A1)
$f(0) = -1.5$ A1
N2

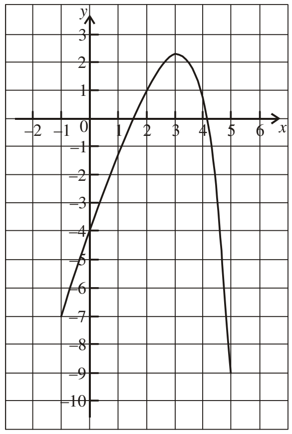
(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ A1A1

N2

[6]

22. (a) intercepts when $f(x) = 0$ (M1)
(1.54, 0) (4.13, 0) (accept $x = 1.54$ $x = 4.13$) A1A1
N3

(b)



A1A1A1
N3

Note: *Award A1 for passing through approximately (0, −4), A1 for correct shape, A1 for a range of approximately –9 to 2.3.*

(c) gradient is 2 A1
N1

[7]

23. (a) (i) $n = 5$ (A1)

$T = 280 \times 1.12^5$

$T = 493$ A1
N2

(ii) evidence of doubling (A1)

e.g. 560

setting up equation A1

e.g. $280 \times 1.12^n = 560$, $1.12^n = 2$

$n = 6.116...$ (A1)

in the year 2007 A1
N3

(b) (i) $P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(5)}}$ (A1)

$P = 39\ 635.993...$ (A1)

$P = 39\ 636$ A1
N3

(ii) $P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(7)}}$

$P = 46\ 806.997...$ A1

not doubled A1
N0

valid reason for **their** answer R1

e.g. $P < 51200$

(c) (i) correct value A2
N2

e.g. $\dfrac{25600}{280}, 91.4, 640{:}7$

(ii) setting up an inequality (accept an equation, or reversed inequality) M1

$e.g. \frac{P}{T} < 70$, $\frac{2\,560\,000}{(10+90e^{-0.1n})280\times1.12^{n}} < 70$

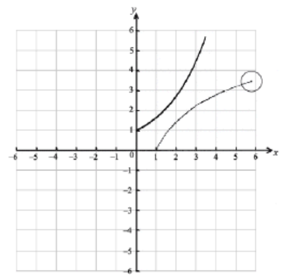
finding the value 9.31.... (A1)

after 10 years A1

N2

[17]

24. (a)



A1A1A1 N3

Note: *Award A1 for approximately correct (reflected) shape, A1 for right end point in circle, A1 for through (1, 0).*

(b) $0 \le y \le 3.5$ A1

N1

(c) interchanging x and y (seen anywhere) M1

e.g. $x = e^{0.5y}$

evidence of changing to log form A1

e.g. $\ln x = 0.5y$, $\ln x = \ln e^{0.5y}$ (any base), $\ln x = 0.5\, y \ln e$ (any base)

$f^{-1}(x) = 2 \ln x$ A1

N1

[7]

25. (a) (i) attempt to substitute (M1)

e.g. $a = \frac{29-15}{2}$

$a = 7$ (accept $a = -7$) A1 N2

(ii) period = 12 (A1)

$b = \frac{2\pi}{12}$ A1

$b = \frac{\pi}{6}$ AG

N0

(iii) attempt to substitute (M1)

e.g. $d = \frac{29+15}{2}$

$d = 22$ A1

N2

(iv) $c = 3$ (accept $c = 9$ from $a = -7$) A1

N1

Note: *Other correct values for c can be found,* $c = 3 \pm 12k$, $k \in \mathbb{Z}$.

(b) stretch takes 3 to 1.5 (A1)

translation maps (1.5, 29) to (4.5, 19) (so M′ is (4.5, 19)) A1

N2

(c) $g(t) = 7\cos\frac{\pi}{3}(t - 4.5) + 12$ A1A2A1

N4

Note: *Award A1 for* $\frac{\pi}{3}$*,* ***A2*** *for 4.5, A1 for 12.*
Other correct values for c can be found
$c = 4.5 \pm 6k,\ k \in \mathbb{Z}$.

(d) translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ (A1)

horizontal stretch of a scale factor of 2 (A1)
completely correct description, in correct order A1
N3

e.g. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ then horizontal stretch of a scale factor of 2

[16]

26. (a) evidence of obtaining the vertex (M1)

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

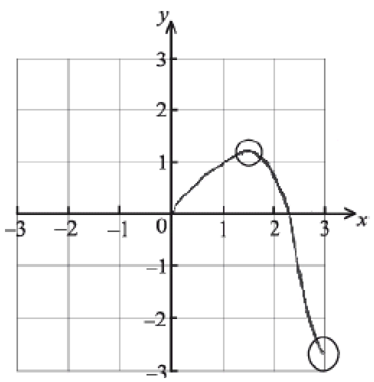
$f(x) = 2(x + 1)^2 - 8$ A2 N3

(b) $x = -1$ (equation must be seen) A1
N1

(c) $f(x) = 2(x - 1)(x + 3)$ A1A1
N2

[6]

27. (a)



A1A2 N3

Notes: *Award* ***A1*** *for correct domain,* $0 \le x \le 3$.
Award ***A2*** *for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.*

(b) $a = 2.31$ A1
N1

(c) evidence of using $V = \pi\int [f(x)]^2\,dx$ (M1)

fully correct integral expression A2

e.g. $V = \pi\int_0^{2.31}[x\cos(x - \sin x)]^2\,dx,\ V = \pi\int_0^{2.31}[f(x)]^2\,dx$

$V = 5.90$ A1

N2

[8]

28. (a) (i) evidence of combining vectors (M1)

e.g. $\vec{AB} = \vec{OB} - \vec{OA}$ (or $\vec{AD} = \vec{AO} + \vec{OD}$ in part (ii))

$\vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ A1

N2

(ii) $\vec{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$ A1

N1

(b) evidence of using perpendicularity $\Rightarrow$ scalar product = 0 (M1)

e.g. $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0$

$4 - 4(k - 5) + 4 = 0$ A1

$-4k + 28 = 0$ (accept any correct equation clearly leading to $k = 7$) A1

$k = 7$ AG

N0

(c) $\vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ (A1)

$\vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ A1

evidence of correct approach (M1)

e.g. $\vec{OC} = \vec{OB} + \vec{BC}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ A1

N3

(d) **METHOD 1**

choosing appropriate vectors, $\vec{BA}$, $\vec{BC}$ (A1)

finding the scalar product M1

e.g. $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$

$\cos A\hat{B}C = 0$ A1

N1

METHOD 2

$\vec{BC}$ parallel to $\vec{AD}$ (may show this on a diagram with points labelled) R1

$\vec{BC} \perp \vec{AB}$ (may show this on a diagram with points labelled) R1

$A\hat{B}C = 90°$

$\cos A\hat{B}C = 0$ A1

N1

[13]

29. $p\boldsymbol{w} = p\boldsymbol{i} + 2p\boldsymbol{j} - 3p\boldsymbol{k}$ (seen anywhere) (A1)

attempt to find $\boldsymbol{v} + p\boldsymbol{w}$ (M1)

e.g. $3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k} + p(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$

collecting terms $(3 + p)\boldsymbol{i} + (4 + 2p)\boldsymbol{j} + (1 - 3p)\boldsymbol{k}$ A1

attempt to find the dot product (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$

setting **their** dot product equal to 0 (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$

simplifying A1

e.g. $3 + p + 8 + 4p - 3 + 9p = 0,\ 14p + 8 = 0$

$P = -0.571 \left(-\frac{8}{14}\right)$ A1

N3

[7]

30. (a) (i) evidence of approach M1

e.g. $\vec{AO} + \vec{OB} = \vec{AB}$, B − A

$\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$ AG

N0

(ii) for choosing **correct** vectors, ($\vec{AO}$ with $\vec{AB}$, or $\vec{OA}$ with $\vec{BA}$) (A1)(A1)

Note: *Using $\overrightarrow{AO}$ with $\overrightarrow{BA}$ will lead to $\pi - 0.799$. If they then say $B\hat{A}O = 0.799$, this is a correct solution.*

calculating $\vec{AO} \bullet \vec{AB}$, $\left|\vec{AO}\right|$, $\left|\vec{AB}\right|$ (A1)(A1)(A1)

e.g. $\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = (-1)(-4) + (2)(6) + (-3)(-1)\ (= 19)$

$|\boldsymbol{d}_1| = \sqrt{(-1)^2 + 2^2 + (-3)^2}\ \left(= \sqrt{14}\right)$,

$|\boldsymbol{d}_2| = \sqrt{(-4)^2 + 6^2 + (-1)^2}\ \left(= \sqrt{53}\right)$

evidence of using the formula to find the angle M1

e.g. $\cos\theta = \dfrac{(-1)(-4) + (2)(6) + (-3)(-1)}{\sqrt{(-1)^2 + 2^2 + (-3)^2}\sqrt{(-4)^2 + 6^2 + (-1)^2}}$,

$\frac{19}{\sqrt{14}\sqrt{53}}, 0.69751...$

$B\hat{A}O = 0.799$ radians (accept 45.8°) A1

N3

(b) two correct answers A1A1

e.g. (1, − 2, 3), (−3, 4, 2), (−7, 10, 1), (−11, 16, 0)

N2

(c) (i) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ A2

N2

(ii) C on L_2, so $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ (M1)

evidence of equating components (A1)

e.g. $1 - 3t = k, -2 + 4t = -k, 5 = 3 + 2t$

one correct value $t = 1, k = -2$ (seen anywhere) (A1)

coordinates of C are (−2, 2, 5) A1

N3

(d) for setting up one (or more) correct equation using

$\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ (M1)

e.g. $3 + p = -2, -8 - 2p = 2, -p = 5$

$p = -5$ A1

N2

[18]

31. evidence of equating vectors (M1)

e.g. $L_1 = L_2$

for any **two** correct equations A1A1

e.g. $2 + s = 3 - t, 5 + 2s = -3 + 3t, 3 + 3s = 8 - 4t$

attempting to solve the equations (M1)

finding **one** correct parameter ($s = -1, t = 2$) A1

the coordinates of T are (1, 3, 0) A1

N3

[6]

32. (a) (i) evidence of approach (M1)

e.g. $\overrightarrow{AO} + \overrightarrow{OB}$, B – A, $\begin{pmatrix} 9-6 \\ -6+2 \\ 15-10 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$ (accept (3, 4, 5)) A1 N2

(ii) evidence of finding the magnitude of the velocity vector M1

e.g. speed = $\sqrt{3^2 + 4^2 + 5^2}$

speed = $\sqrt{50}$ $(=5\sqrt{2})$ A1

N1

(b) correct **equation** (accept Cartesian and parametric forms) A2

N2

e.g. $\boldsymbol{r} = \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}, \boldsymbol{r} = \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

[6]

33. (a) evidence of addition (M1)

e.g. at least two correct elements

$\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}$ A1

N2

(b) evidence of multiplication (M1)

e.g. at least two correct elements

$-3\boldsymbol{A} = \begin{pmatrix} -3 & -6 \\ -9 & 3 \end{pmatrix}$ A1

N2

(c) evidence of matrix multiplication (in correct order) (M1)

e.g. $\boldsymbol{AB} = \begin{pmatrix} 1(3)+2(-2) & 1(0)+2(1) \\ 3(3)+(-1)(-2) & 3(0)+(-1)(1) \end{pmatrix}$

$\boldsymbol{AB} = \begin{pmatrix} -1 & 2 \\ 11 & -1 \end{pmatrix}$ A2

N3

[7]

34. (a) det $\boldsymbol{M} = -4$ A1 N1

(b) $\boldsymbol{M}^{-1} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right)$ A1A1

N2

Note: *Award A1 for* $-\frac{1}{4}$ *and A1 for the correct matrix.*

(c) $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \left(\boldsymbol{X} = -\frac{1}{4}\begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 8 \end{pmatrix} \right)$ M1

$\boldsymbol{X} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $(x=3, y=-2)$ A1A1

N0

Note: *Award no marks for an* ***algebraic*** *solution of the system $2x + y = 4$, $2x - y = 8$.*

[6]

35. (a) evidence of correct method (M1)

e.g. at least 1 correct element (must be in a 2 × 2 matrix)

$\boldsymbol{AB} = \begin{pmatrix} -2-2q & 0 \\ -6+pq & 3+\frac{p}{2} \end{pmatrix}$ A1 N2

(b) **METHOD 1**

evidence of using $\boldsymbol{AB} = \boldsymbol{I}$ (M1)

2 correct equations A1A1

e.g. $-2 - 2q = 1$ and $3 + \frac{p}{2} = 1$, $-6 + pq = 0$

$p = -4$, $q = -\frac{3}{2}$ A1A1

N1N1

METHOD 2

finding $\boldsymbol{A}^{-1} = \frac{1}{p+6}\begin{pmatrix} p & 2 \\ -3 & 1 \end{pmatrix}$ A1

evidence of using $\boldsymbol{A}^{-1} = \boldsymbol{B}$ (M2)

e.g. $\frac{2}{p+6} = 1$ and $-\frac{3}{p+6} = q$, $\frac{p}{p+6} = -2$ and $-\frac{3}{p+6} = q$

$p = -4$, $q = -\frac{3}{2}$ A1A1

N1N1

[7]

36. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & -0.5 & 1.25 \\ 1 & -0.5 & 0.75 \end{pmatrix}$ A2 N2

(b) (i) $\boldsymbol{I} - \frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1}$ A1

$-\frac{1}{2}\boldsymbol{B} = \boldsymbol{A}^{-1} - \boldsymbol{I}$ A1

$\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$ AG

(ii) $\boldsymbol{B} = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 3 & -2.5 \\ -2 & 1 & 0.5 \end{pmatrix}$ A2

N2

(iii) $\det \boldsymbol{B} = 12$ A1

N1

(iv) $\det \boldsymbol{B} \neq 0$ R1

N1

(c) (i) evidence of using a valid approach M1

e.g. $\boldsymbol{X} = \boldsymbol{B}^{-1}\boldsymbol{C}$

$\boldsymbol{X} = \begin{pmatrix} 0.333 \\ 1 \\ 1.33 \end{pmatrix} \left(= \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{4}{3} \end{pmatrix} \right)$ A1

N1

(ii) $4x - 2y + 2z = 2$, $-2x + 3y - 2.5z = -1$, $-2x + y + 0.5z = 1$ A1A1A1

N3

[13]