Final Exam Prep – Practice Problems

1. Consider the arithmetic sequence 2, 5, 8, 11, ....
   (a) Find \( u_{101} \).
   
   (3)

   (b) Find the value of \( n \) so that \( u_n = 152 \).

   (3)

   (Total 6 marks)

2. Consider the infinite geometric sequence 3000, –1800, 1080, –648, ....
   (a) Find the common ratio.

   (2)

   (b) Find the 10th term.

   (2)

   (c) Find the exact sum of the infinite sequence.

   (2)

   (Total 6 marks)

3. Find the term in \( x^3 \) in the expansion of \( \left( \frac{2}{3} x^3 - 3 \right)^6 \).

   (Total 5 marks)

4. Consider the infinite geometric sequence 3, 3(0.9), 3(0.9)^2, 3(0.9)^3, ....
   (a) Write down the 10th term of the sequence. Do not simplify your answer.

   (1)

   (b) Find the sum of the infinite sequence.

   (4)

   (Total 5 marks)

5. (a) Expand \( (x - 2)^d \) and simplify your result.

   (3)

   (b) Find the term in \( x^3 \) in \( (3x + 4)(x - 2)^d \).

   (3)

   (Total 6 marks)

6. Let \( f (x) = \ln (x + 5) + \ln 2 \), for \( x > -5 \).
   (a) Find \( f^{-1} (x) \).

   (4)

   Let \( g (x) = e^x \).

   (b) Find \( (g \circ f) (x) \), giving your answer in the form \( ax + b \), where \( a, b \in \mathbb{Z} \).

   (3)

   (Total 7 marks)

7. Let \( f (x) = 3(x + 1)^2 - 12 \).
   (a) Show that \( f (x) = 3x^2 + 6x - 9 \).

   (2)

   (b) For the graph of \( f \)

   (i) write down the coordinates of the vertex;
   (ii) write down the equation of the axis of symmetry;
   (iii) write down the \( y \)-intercept;
   (iv) find both \( x \)-intercepts.

   (8)

   (c) Hence sketch the graph of \( f \).

   (2)

   (d) Let \( g (x) = x^2 \). The graph of \( f \) may be obtained from the graph of \( g \) by the two transformations:

   a stretch of scale factor \( t \) in the \( y \)-direction
   followed by a translation of \( \left( \frac{p}{q} \right) \)

   Find \( \left( \frac{p}{q} \right) \) and the value of \( t \).

   (3)

   (Total 15 marks)
8. Consider \( f(x) = \sqrt{x-5} \).
   (a) Find
   (i) \( f(11) \);
   (ii) \( f(86) \);
   (iii) \( f(5) \).
   (b) Find the values of \( x \) for which \( f \) is undefined.
   (c) Let \( g(x) = x^2 \). Find \( (g \circ f)(x) \).

9. The quadratic function \( f \) is defined by \( f(x) = 3x^2 - 12x + 11 \).
   (a) Write \( f \) in the form \( f(x) = 3(x-h)^2 - k \).
   (b) The graph of \( f \) is translated 3 units in the positive \( x \)-direction and 5 units in the positive \( y \)-direction. Find the function \( g \) for the translated graph, giving your answer in the form \( g(x) = 3(x-p)^2 + q \).

10. Let \( M = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} \), and \( O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \). Given that \( M^2 - 6M + kI = O \), find \( k \).

11. Let \( f(x) = 2x^2 - 12x + 5 \).
   (a) Express \( f(x) \) in the form \( f(x) = 2(x-h)^2 - k \).
   (b) Write down the vertex of the graph of \( f \).
   (c) Write down the equation of the axis of symmetry of the graph of \( f \).
   (d) Find the \( y \)-intercept of the graph of \( f \).
   (e) The \( x \)-intercepts of \( f \) can be written as \( \frac{p \pm \sqrt{q}}{r} \), where \( p, q, r \in \mathbb{Z} \).

12. Let \( f(x) = \frac{1}{x} \), \( x \neq 0 \).
   (a) Sketch the graph of \( f \).
   (b) Find an expression for \( g(x) \).
   (c) (i) Find the intercepts of \( g \).
       (ii) Write down the equations of the asymptotes of \( g \).
       (iii) Sketch the graph of \( g \).
13. The function \( f \) is defined by \( f(x) = \frac{3}{\sqrt{9-x^2}} \), for \(-3 < x < 3\). (Calculator OK)

(a) On the grid below, sketch the graph of \( f \).

(b) Write down the equation of each vertical asymptote.

(c) Write down the range of the function \( f \).

(Total 6 marks)

14. The functions \( f \) and \( g \) are defined by \( f : x \mapsto 3x \), \( g : x \mapsto x + 2 \).

(a) Find an expression for \( (f \circ g)(x) \).

(b) Find \( f^{-1}(18) + g^{-1}(18) \).

(Total 6 marks)

15. The following diagram shows part of the graph of \( f \), where \( f(x) = x^2 - x - 2 \).

(a) Find both \( x \)-intercepts.

(b) Find the \( x \)-coordinate of the vertex.

(Total 6 marks)
16. In an arithmetic sequence \( u_{21} = -37 \) and \( u_4 = -3 \).
   (a) Find
      (i) the common difference;
      (ii) the first term.
   (b) Find \( S_{10} \).

17. Let \( u_n = 3 - 2n \).
   (a) Write down the value of \( u_1, u_2, \) and \( u_3 \).
   (b) Find \( \sum_{n=1}^{20} (3 - 2n) \).

18. Solve the following equations.
   (a) \( \log x 49 = 2 \)
   (b) \( \log_2 8 = x \)
   (c) \( \log_{25} x = -\frac{1}{2} \)
   (d) \( \log_2 x + \log_2(x - 7) = 3 \)

19. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.
   (a) Calculate the number of seats in the 20th row.
   (b) Calculate the total number of seats.

20. A sum of $5000 is invested at a compound interest rate of 6.3 % per annum.
   (a) Write down an expression for the value of the investment after \( n \) full years.
   (b) What will be the value of the investment at the end of five years?
   (c) The value of the investment will exceed $10 000 after \( n \) full years.
      (i) Write down an inequality to represent this information.
      (ii) Calculate the minimum value of \( n \).
21. Part of the graph of a function \( f \) is shown in the diagram below.

(a) On the same diagram sketch the graph of \( y = -f(x) \).

(b) Let \( g(x) = f(x + 3) \).
   (i) Find \( g(-3) \).
   (ii) Describe fully the transformation that maps the graph of \( f \) to the graph of \( g \).

22. Let \( f(x) = 3x - e^{x-2} - 4 \), for \(-1 \leq x \leq 5\).
   (a) Find the \( x \)-intercepts of the graph of \( f \).
   (b) On the grid below, sketch the graph of \( f \).
   (c) Write down the gradient of the graph of \( f \) at \( x = 2 \).
23. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After $n$ years the number of taxis, $T$, in the city is given by

$$T = 280 \times 1.12^n.$$  

(a) (i) Find the number of taxis in the city at the end of 2005.  
(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.  

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After $n$ years the number of people, $P$, in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90e^{-0.1n}}.$$  

(i) Find the value of $P$ at the end of 2005, giving your answer to the nearest whole number.  
(ii) After seven complete years, will the value of $P$ be double its value at the end of 2000? Justify your answer.  

(c) Let $R$ be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.  
(i) Find the value of $R$ at the end of 2000.  
(ii) After how many complete years will the city first reduce the number of taxis?  

(Total 17 marks)

24. Let $f$ be the function given by $f(x) = e^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of $f$.

(a) On the same diagram, sketch the graph of $f^{-1}$.  
(b) Write down the range of $f^{-1}$.  
(c) Find $f^{-1}(x)$.  

(Total 7 marks)
25. [Note: Trig Functions will not be included on the final exam. But transformations of functions will be. Do this problem only if you want to explore transformations more thoroughly.]

Let \( f(t) = a \cos b (t - c) + d, t \geq 0 \). Part of the graph of \( y = f(t) \) is given below.

When \( t = 3 \), there is a maximum value of 29, at M.
When \( t = 9 \), there is a minimum value of 15.

(a) (i) Find the value of \( a \).
(ii) Show that \( b = \frac{\pi}{6} \).
(iii) Find the value of \( d \).
(iv) Write down a value for \( c \).

The transformation \( P \) is given by a horizontal stretch of a scale factor of \( \frac{1}{2} \), followed by a translation of \( \begin{pmatrix} 3 \\ -10 \end{pmatrix} \).

(b) Let \( M' \) be the image of \( M \) under \( P \). Find the coordinates of \( M' \).

The graph of \( g \) is the image of the graph of \( f \) under \( P \).

(c) Find \( g(t) \) in the form \( g(t) = 7 \cos B(t - C) + D \).

(d) Give a full geometric description of the transformation that maps the graph of \( g \) to the graph of \( f \).

26. Let \( f(x) = 2x^2 + 4x - 6 \).

(a) Express \( f(x) \) in the form \( f(x) = 2(x - h)^2 + k \).

(b) Write down the equation of the axis of symmetry of the graph of \( f \).

(c) Express \( f(x) \) in the form \( f(x) = 2(x - p)(x - q) \).

27. Let \( f(x) = x \cos (x - \sin x), 0 \leq x \leq 3 \).

(a) Sketch the graph of \( f \) on the following set of axes.
(b) The graph of \( f \) intersects the \( x \)-axis when \( x = a, a \neq 0 \). Write down the value of \( a \).

28. Consider the points \( A (1, 5, 4) \), \( B (3, 1, 2) \) and \( D (3, k, 2) \), with \( (AD) \) perpendicular to \( (AB) \).
   (a) Find
      (i) \( \overrightarrow{AB} \);
      (ii) \( \overrightarrow{AD} \), giving your answer in terms of \( k \).
   (b) Show that \( k = 7 \).

   The point \( C \) is such that \( \overrightarrow{BC} = \frac{1}{2} \overrightarrow{AD} \).

   (c) Find the position vector of \( C \).

   (d) Find \( \cos \angle ABC \).

29. Let \( v = 3i + 4j + k \) and \( w = i + 2j - 3k \). The vector \( v + pw \) is perpendicular to \( w \).

   Find the value of \( p \).

30. The point \( O \) has coordinates \( (0, 0, 0) \), point \( A \) has coordinates \( (1, -2, 3) \) and point \( B \) has coordinates \( (-3, 4, 2) \).

   (a) (i) Show that \( \overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} \).
   (ii) Find \( \overrightarrow{BA} \).

   (b) The line \( L_1 \) has equation \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} \).

   Write down the coordinates of two points on \( L_1 \).

   (c) The line \( L_2 \) passes through \( A \) and is parallel to \( \overrightarrow{OB} \).
      (i) Find a vector equation for \( L_2 \), giving your answer in the form \( \mathbf{r} = \mathbf{a} + t\mathbf{b} \).
      (ii) Point \( C \) \( (k, -k, 5) \) is on \( L_2 \). Find the coordinates of \( C \).
(d) The line \( L_3 \) has equation \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \), and passes through the point C.

Find the value of \( p \) at C.

(2)

(Total 18 marks)

31. The line \( L_1 \) is represented by \( r_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \) and the line \( L_2 \) by \( r_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} \).

The lines \( L_1 \) and \( L_2 \) intersect at point T. Find the coordinates of T.

(Total 6 marks)

32. A particle is moving with a constant velocity along line \( L \). Its initial position is A(6, -2, 10). After one second the particle has moved to B(9, -6, 15).

(a) (i) Find the velocity vector, \( AB \).

(ii) Find the speed of the particle.

(b) Write down an equation of the line \( L \).

(4)

(2)

(Total 6 marks)

33. Let \( A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \) and \( B = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix} \).

Find

(a) \( A + B \); \hspace{1cm} (2)

(b) \( -3A \); \hspace{1cm} (2)

(c) \( AB \). \hspace{1cm} (3)

(Total 7 marks)

34. Let \( M = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} \).

(a) Write down the determinant of \( M \). \hspace{1cm} (1)

(b) Write down \( M^{-1} \). \hspace{1cm} (2)

(c) Hence solve \( M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \). \hspace{1cm} (3)

(Total 6 marks)

35. Let \( A = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix} \) and \( B = \begin{pmatrix} -2 & 1 \\ q & 1/2 \end{pmatrix} \).

(a) Find \( AB \) in terms of \( p \) and \( q \). \hspace{1cm} (2)

(b) Matrix \( B \) is the inverse of matrix \( A \). Find the value of \( p \) and of \( q \). \hspace{1cm} (5)
36. Let  
\[
A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}
\]

(a) Write down  \(A^{-1}\).

The matrix  \(B\) satisfies the equation  \(\left( I - \frac{1}{2} B \right)^{-1} = A\), where  \(I\) is the 3 \(\times\) 3 identity matrix.

(b) (i) Show that  \(B = -2(A^{-1} - I)\).
(ii) Find  \(B\).
(iii) Write down  \(\text{det}\ B\).
(iv) Hence, explain why  \(B^{-1}\) exists.

Let  \(BX = C\), where  \(X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}\) and  \(C = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\).

(c) (i) Find  \(X\).
(ii) Write down a system of equations whose solution is represented by  \(X\).
Final Exam Prep – Practice Problems: MarkScheme

1. (a) \[ d = 3 \]  
   evidence of substitution into \( u_n = a + (n - 1) \cdot d \)  
   \( e.g. \ u_{101} = 2 + 100 \times 3 \)  
   \( u_{101} = 302 \)  
   (A1)  
   (M1)  
   A1  
   N3  

   (b) correct approach  
   \( e.g. \ 152 = 2 + (n - 1) \times 3 \)  
   correct simplification  
   \( e.g. \ 150 = (n - 1) \times 3, 50 = n - 1, 152 = -1 + 3n \)  
   \( n = 51 \)  
   A1  
   N2

2. (a) evidence of dividing two terms  
   \( e.g. \ \frac{1800}{3000}, -\frac{1800}{1080} \)  
   \( r = -0.6 \)  
   A1  
   N2

   (b) evidence of substituting into the formula for the 10th term  
   \( e.g. \ u_{10} = 3000(-0.6)^9 \)  
   \( u_{10} = -30.2 \)  
   (accept the exact value \(-30.233088\))  
   A1  
   N2

   (c) evidence of substituting into the formula for the infinite sum  
   \( e.g. \ S = \frac{3000}{1.6} \)  
   \( S = 1875 \)  
   A1  
   N2

3. evidence of using binomial expansion  
   \( e.g. \ \text{selecting correct term, } a^8b^0 + \binom{8}{1}a^7b + \frac{8}{2}a^6b^2 + ... \)  
   evidence of calculating the factors, in any order  
   \( e.g. \ 56, \frac{2^3}{3^3}, -3^5, \left(\frac{8}{5}\right)\left(\frac{2}{3}\right)^3 (-3)^5 \)  
   \(-4032x^3\)  
   (accept \(-4030x^3\) to 3 s.f.)  
   A1  
   N2

4. (a) \( u_{10} = 3(0.9)^9 \)  
   A1  
   N1

   (b) recognizing \( r = 0.9 \)  
   correct substitution  
   \( e.g. \ S = \frac{3}{1-0.9} \)  
   \( S = \frac{3}{0.1} \)  
   \( S = 30 \)  
   A1  
   A1  
   N3

5. (a) evidence of expanding  
   \( e.g. \ (x - 2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4 \)  
   M1

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\[(x - 2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16\]  
\[\text{(b) finding coefficients, } 3 \times 24 = 72, 4 \times (-8) = -32\]  
\[\text{term is } 40x^3\]  
\[\text{(A1)(A1)}\]  

6. (a) \textbf{METHOD 1}
\[\ln (x + 5) + \ln 2 = \ln (2(x + 5)) (= \ln (2x + 10))\]  
\[\text{interchanging } x \text{ and } y \text{ (seen anywhere)}\]  
\[\text{e.g. } x = \ln (2y + 10)\]  
\[\text{evidence of correct manipulation}\]  
\[\text{e.g. } e^x = 2y + 10\]  
\[f^{-1}(x) = \frac{e^x - 10}{2}\]  
\[\text{N2}\]  

\textbf{METHOD 2}
\[y = \ln (x + 5) + \ln 2\]  
\[y - \ln 2 = \ln (x + 5)\]  
\[\text{evidence of correct manipulation}\]  
\[\text{e.g. } e^{y - \ln 2} = x + 5\]  
\[\text{interchanging } x \text{ and } y \text{ (seen anywhere)}\]  
\[\text{e.g. } e^{y - \ln 2} = y + 5\]  
\[f^{-1}(x) = e^{x - \ln 2} - 5\]  
\[\text{N2}\]  

(b) \textbf{METHOD 1}
\[\text{evidence of composition in correct order}\]  
\[\text{e.g. } (g \circ f)(x) = g(\ln (x + 5) + \ln 2)\]  
\[= e^{\ln (2(x + 5))} = 2(x + 5)\]  
\[(g \circ f)(x) = 2x + 10\]  
\[\text{N2}\]  

\textbf{METHOD 2}
\[\text{evidence of composition in correct order}\]  
\[\text{e.g. } (g \circ f)(x) = e^{\ln (x + 5) + \ln 2}\]  
\[= e^{\ln (x + 5)} \times \ln 2 = (x + 5) \times 2\]  
\[(g \circ f)(x) = 2x + 10\]  
\[\text{N2}\]  

7. (a) \[f(x) = 3(x^2 + 2x + 1) - 12\]  
\[= 3x^2 + 6x + 3 - 12\]  
\[= 3x^2 + 6x - 9\]  
\[\text{A1}\]  

(b) (i) \text{vertex is } (-1, -12)\]  
\[\text{A1}\]  

(ii) \[x = -1 \text{ (must be an equation)}\]  
\[\text{A1}\]  

(iii) \[0, -9\]  
\[\text{A1}\]  

(iv) \text{evidence of solving } f(x) = 0\]  
\[\text{e.g. factorizing, formula, correct working}\]  
\[\text{A1}\]
\[
3(x + 3)(x - 1) = 0, \quad x = \frac{-6 \pm \sqrt{36 + 108}}{6} = (-3, 0), (1, 0)
\]

(c) 

\[
\begin{aligned}
&y \quad \downarrow \\
&\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
&-12 \quad -5 \quad 1 \quad 6
\end{aligned}
\]

Notes: Award A1 for a parabola opening upward.
A1 for vertex and intercepts in approximately correct positions.

(d) \[
\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}, \quad t = 3 \quad \text{(accept } p = -1, q = -12, t = 3)\]

8. (a) (i) \(\sqrt{6}\) (A1 N1)
(ii) 9 (A1)
(iii) 0 (A1)

(b) \(x < 5\) (A2)

(c) \((g \circ f)(x) = (\sqrt{x - 5})^2 = x - 5\) (M1)

9. (a) For a reasonable attempt to complete the square, (or expanding) (M1) 
\(e.g. 3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12\) 
f(x) = 3(x - 2)^2 - 1 (accept h = 2, k = 1) (A1 A1 N3)

(b) METHOD 1
Vertex shifted to \((2 + 3, -1 + 5) = (5, 4)\) 
so the new function is \(3(x - 5)^2 + 4\) (accept \(p = 5, q = 4\)) (A1 A1 N2)

METHOD 2
\(g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5\) 
\(= 3(x - 5)^2 + 4\) (accept \(p = 5, q = 4\)) (A1 A1 N2)

10. \[
\begin{pmatrix}
2 & -1 \\
-3 & 4
\end{pmatrix}
\begin{pmatrix}
2 & -1 \\
-3 & 4
\end{pmatrix}
\begin{pmatrix}
-6 \quad -1 \\
-3 & 4
\end{pmatrix}
+ k\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
\]
\[ M^2 = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix} \quad \text{A2} \]

\[ 6M = \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix} \quad \text{A1} \]

\[ \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{A1} \]

\[ k = 5 \quad \text{A1} \]

\[ \text{N2} \]

\[ \text{[6]} \]

11. (a) Evidence of completing the square

\[ f(x) = 2(x^2 - 6x + 9) + 5 - 18 \]

\[ = 2(x - 3)^2 - 13 \quad \text{(accept } h = 3, k = 13) \quad \text{A1} \]

\[ \text{N3} \]

(b) Vertex is (3, -13) \quad \text{A1A1} \]

\[ \text{N2} \]

(c) \( x = 3 \) (must be an equation) \quad \text{A1} \]

\[ \text{N1} \]

(d) evidence of using fact that \( x = 0 \) at \( y \)-intercept

\[ y \)-intercept is (0, 5) (accept 5) \quad \text{A1} \]

\[ \text{N2} \]

(e) **METHOD 1**

evidence of using \( y = 0 \) at \( x \)-intercept

\[ e.g. \ 2(x - 3)^2 - 13 = 0 \]

\[ \text{evidence of solving this equation} \quad \text{M1} \]

\[ e.g. \ (x - 3)^2 = \frac{13}{2} \quad \text{A1} \]

\[ (x - 3) = \pm \sqrt{\frac{13}{2}} \]

\[ x = 3 \pm \sqrt{\frac{13}{2}} = 3 \pm \sqrt{\frac{26}{2}} \quad \text{A1} \]

\[ x = \frac{6 \pm \sqrt{26}}{2} \]

\[ p = 6, q = 26, r = 2 \quad \text{A1A1A1} \]

\[ \text{N4} \]

**METHOD 2**

evidence of using \( y = 0 \) at \( x \)-intercept

\[ e.g. \ 2x^2 - 12x + 5 = 0 \]

\[ \text{evidence of using the quadratic formula} \quad \text{M1} \]

\[ x = \frac{12 \pm \sqrt{144 - 4 \times 2 \times 5}}{2 \times 2} \quad \text{A1} \]

\[ x = \frac{12 \pm \sqrt{104}}{4} \left( = \frac{6 \pm \sqrt{26}}{2} \right) \quad \text{A1} \]

\[ p = 12, q = 104, r = 4 \quad \text{(or } p = 6, q = 26, r = 2) \quad \text{A1A1A1} \]

\[ \text{N4} \]

\[ \text{[15]} \]
12. (a) 

Note: Award A1 for the left branch, and A1 for the right branch.

(b) \[ g(x) = \frac{1}{x-2} + 3 \]

(c) (i) Evidence of using \( x = 0 \) \( \left( g(0) = -\frac{1}{2} + 3 \right) \)

\[ y = \frac{5}{2} \ (= 2.5) \]

Evidence of solving \( y = 0 \) \((1 + 3(x-2) = 0)\)

\[ 1 + 3x - 6 = 0 \]

\[ 3x = 5 \]

\[ x = \frac{5}{3} \]

Intercepts are \( x = \frac{5}{3} \), \( y = \frac{5}{2} \) (accept \( \left( \frac{5}{3}, 0 \right) \) \( \left( 0, \frac{5}{2} \right) \))

(ii) \( x = 2 \)

\( y = 3 \)

(iii) 

Note: Award A1 for the shape (both branches), A1 for the correct behaviour close to the asymptotes, and A1 for the intercepts at approximately \( \left( \frac{5}{3}, 0 \right) \) \( \left( 0, \frac{5}{2} \right) \).
13. (a) 

![Graph of a function](image)

**Note:** Award A1 for the general shape and A1 for the y-intercept at 1.

(b) \( x = 3, x = -3 \)  A1A1

(c) \( y \geq 1 \)  N1N1

14. (a) \((f \circ g): x \mapsto 3(x + 2) \)(= 3\(x + 6\))  A2

(b) **METHOD 1**

Evidence of finding inverse functions  M1

\[ e.g. f^{-1}(x) = \frac{x}{3}, \quad g^{-1}(x) = x - 2 \]

\[ f^{-1}(18) = \frac{18}{3} = 6 \] (A1)

\[ g^{-1}(18) = 18 - 2 = 16 \] (A1)

\[ f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22 \] A1

**METHOD 2**

Evidence of solving equations  M1

\[ e.g. 3x = 18, x + 2 = 18 \]

\[ x = 6, x = 16 \] (A1)(A1)

\[ f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22 \] A1

15. (a) evidence of attempting to solve \( f(x) = 0 \)  (M1)

Evidence of correct working  A1

\[ e.g. \ (x+1)(x-2), \ \frac{1 \pm \sqrt{9}}{2} \]

intercepts are \((-1, 0)\) and \((2, 0)\) (accept \(x = -1, x = 2\))  A1A1

(b) evidence of appropriate method  (M1)

\[ e.g. \ x_v = \frac{x_1 + x_2}{2}, \ x_v = -\frac{b}{2a}, \ \text{reference to symmetry} \]

\[ x_v = 0.5 \] A1

16. (a) (i) attempt to set up equations  (M1)

\[ -37 = u_1 + 20d \text{ and } -3 = u_1 + 3d \]  A1

\[ -34 = 17d \]

\[ d = -2 \] A1 N2
(ii) \(-3 = u_1 - 6 \Rightarrow u_1 = 3\)  
\[u_1 = 3\]  
A1  
N1  

(b)  
\[u_{10} = 3 + 9 \times (-2) = -15\]  
\[A1\]  
\[S_{10} = \frac{10}{2} (3 + (-15))\]  
\[M1\]  
\[= -60\]  
A1  
N2

17. (a) \[u_1 = 1, u_2 = -1, u_3 = -3\]  
A1 A1 A1  
N3  

(b) Evidence of using appropriate formula  
M1  
Correct values \(S_{20} = \frac{20}{2} (2 \times 1 + 19 \times -2) (= 10(2 - 38))\)  
A1  
\[S_{20} = -360\]  
A1  
N1  

18. (a)  
\[x^2 = 49\]  
\[M1\]  
\[x = \pm 7\]  
A1  
N3  
\[x = 7\]  
A1  
N3  

(b)  
\[2^x = 8\]  
\[M1\]  
\[x = 3\]  
A1  
N2  

(c)  
\[x = 25\frac{1}{2}\]  
\[M1\]  
\[x = \frac{1}{\sqrt{25}}\]  
\[A1\]  
\[x = \frac{1}{5}\]  
A1  
N3

(d)  
\[\log_2 (x(x - 7)) = 3\]  
\[M1\]  
\[\log_2 (x^2 - 7x) = 3\]  
\[A1\]  
\[2^3 = 8 \quad (8 = x^2 - 7x)\]  
\[A1\]  
\[x^2 - 7x - 8 = 0\]  
\[A1\]  
\[(x - 8)(x + 1) = 0 \quad (x = 8, x = -1)\]  
\[A1\]  
\[x = 8\]  
A1  
N3

19. (a) Recognizing an AP  
M1  
\[u_1 = 15, d = 2, n = 20\]  
\[A1\]  
Substituting into \(u_{20} = 15 + (20 - 1) \times 2\)  
M1  
= 53 (that is, 53 seats in the 20th row)  
A1  
N2  

(b) Substituting into \(S_{20} = \frac{20}{2} (2(15) + (20 - 1)2)\) (or into \(\frac{20}{2} (15 + 53)\))  
M1  
= 680 (that is, 680 seats in total)  
A1  
N2

20. (a)  
\[5000(1.063)^n\]  
\[A1\]  
\[N1\]  

(b) Value = $5000(1.063)^5 (= $6786.3511...)  
\[= $6790 to 3 s.f. (accept $6786, or $6786.35)\]  
\[A1\]  
\[N1\]
(c) (i) \[5000(1.063)^n > 10000 \text{ or } (1.063)^n > 2\]  
\[\text{A1} \]  
\[\text{N1} \]  

(ii) Attempting to solve the inequality \[n \log(1.063) > \log 2\]  
\[n > 11.345\]  
\[\text{A1} \]  
\[\text{A1} \]  
\[\text{N3} \]  

\textbf{Note:} Candidates are likely to use TABLE or LIST on a GDC to find \(n\).  
A good way of communicating this is suggested below.  

Let \(y = 1.063^x\)  
When \(x = 11, y = 1.9582,\) when \(x = 12, y = 2.0816\)  
\(x = 12\) i.e. 12 years  
\[\text{A1} \]  
\[\text{N3} \]  

21. (a)  

![Graph](image_url)  

\textbf{Note:} Award M1 for evidence of reflection in \(x\)-axis, A1 for correct vertex and all intercepts approximately correct.  

(b) (i) \(g(-3) = f(0)\)  
\(f(0) = -1.5\)  
\[\text{A1} \]  
\[\text{A1} \]  
\[\text{N2} \]  

(ii) translation (accept shift, slide, etc.) of \[
\begin{pmatrix}
-3 \\
0
\end{pmatrix}
\]  
\[\text{A1A1} \]  
\[\text{N2} \]  

22. (a) intercepts when \(f(x) = 0\)  
\(1.54, 0\) (4.13, 0) (accept \(x = 1.54\) \(x = 4.13\))  
\[\text{M1} \]  
\[\text{A1A1} \]  
\[\text{N3} \]  

(b)
23. (a) (i) \( n = 5 \) 
\[ T = 280 \times 1.12^5 \]
\[ T = 493 \]  
(A1) 
A1 
N2

(ii) evidence of doubling 
\( e.g. \ 560 \) 
setting up equation 
\( e.g. \ 280 \times 1.12^n = 560, 1.12^n = 2 \) 
n = 6.116... 
in the year 2007  
(A1) 
A1 
N3

(b) (i) \[ P = \frac{2560000}{10 + 90 e^{-0.1(5)}} \]  
P = 39 635.993...  
(A1) 
A1 
N3

(ii) \[ P = \frac{2560000}{10 + 90 e^{-0.1(7)}} \]  
P = 46 806.997...  
not doubled  
(A1) 
A1 
N0

valid reason for their answer 
\( e.g. \ P < 51200 \)  
R1

(c) (i) correct value 
\( e.g. \ \frac{25600}{280}, 91.4, 640:7 \)  
A2 
N2

(ii) setting up an inequality (accept an equation, or reversed inequality)  
M1
\[ e.g. \frac{P}{T} < 70, \quad \frac{2560000}{(10 + 90e^{-0.1t})280 \times 1.12^n} < 70 \]

finding the value 9.31.... after 10 years

24. (a)

\[ \text{Note:} \quad \text{Award A1 for approximately correct (reflected) shape,}\]
\[ \text{A1 for right end point in circle, A1 for through (1, 0).} \]

(b) \[ 0 \leq y \leq 3.5 \]

(c) interchanging \( x \) and \( y \) (seen anywhere)

\[ e.g. x = e^{0.5y} \]

\[ \text{evidence of changing to log form} \]
\[ e.g. \ ln x = 0.5y, \ ln x = \ln e^{0.5y} \] (any base), \( \ln x = 0.5 \ln e \) (any base)
\[ f^{-1}(x) = 2 \ln x \]

25. (a) (i) attempt to substitute

\[ e.g. a = \frac{29 - 15}{2} \]
\[ a = 7 \] (accept \( a = -7 \))

(ii) period = 12

\[ b = \frac{2\pi}{12} \]
\[ b = \frac{\pi}{6} \]

(iii) attempt to substitute

\[ e.g. d = \frac{29 + 15}{2} \]
\[ d = 22 \]

(iv) \( c = 3 \) (accept \( c = 9 \) from \( a = -7 \))

\[ \text{Note:} \quad \text{Other correct values for} \ c \ \text{can be found,} \]
\[ c = 3 \pm 12k, k \in \mathbb{Z}. \]

(b) stretch takes 3 to 1.5

translation maps (1.5, 29) to (4.5, 19) (so \( M' \) is (4.5, 19))
(c) \(g(t) = 7 \cos \frac{\pi}{3} (t - 4.5) + 12\)  

\[\text{Note: Award A1 for } \frac{\pi}{3}, \ A2 \text{ for } 4.5, \ A1 \text{ for } 12.\]
\[\text{Other correct values for } c \text{ can be found } c = 4.5 \pm 6k, k \in \mathbb{Z}.\]

(d) translation \(\begin{pmatrix} -3 \\ 10 \end{pmatrix}\)  
horizontal stretch of a scale factor of 2  
completely correct description, in correct order  
e.g. translation \(\begin{pmatrix} -3 \\ 10 \end{pmatrix}\) then horizontal stretch of a scale factor of 2

26. (a) evidence of obtaining the vertex   
e.g. a graph, \(x = -\frac{b}{2a}\), completing the square  
\(f(x) = 2(x + 1)^2 - 8\)  
(b) \(x = -1\) (equation must be seen)  
(c) \(f(x) = 2(x - 1)(x + 3)\)

27. (a) evidence of using \(V = \pi \int_0^{2.31} [f(x)]^2 \, dx\)  
fully correct integral expression  
e.g. \(V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 \, dx, V = \pi \int_0^{2.31} [f(x)]^2 \, dx\)  
Notes: Award A1 for correct domain, \(0 \leq x \leq 3\).  
Award A2 for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.

(b) \(a = 2.31\)

(c) evidence of obtaining the vertex   
e.g. a graph, \(x = -\frac{b}{2a}\), completing the square  
\(f(x) = 2(x + 1)^2 - 8\)  
(b) \(x = -1\) (equation must be seen)  
(c) \(f(x) = 2(x - 1)(x + 3)\)
28. (a) (i) evidence of combining vectors (M1)
   \[ \vec{AB} = \vec{OB} - \vec{OA} \] (or \[ \vec{AD} = \vec{AO} + \vec{OD} \] in part (ii))
   \[ \vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \]
   A1
   (ii) \[ \vec{AD} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} \]
   A1
   (b) evidence of using perpendicularity ⇒ scalar product = 0 (M1)
   \[ \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} = 0 \]
   \[ 4 - 4(k-5) + 4 = 0 \]
   \[ -4k + 28 = 0 \] (accept any correct equation clearly leading to \( k = 7 \))
   \( k = 7 \) AG
   N0
   (c) \[ \vec{AD} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \]
   (A1)
   \[ \vec{BC} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \]
   A1
   evidence of correct approach (M1)
   \[ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x-3 \\ y-1 \\ z-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \]
   \[ \vec{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} \]
   A1
   (d) METHOD 1
   choosing appropriate vectors, \( \vec{BA}, \vec{BC} \) (A1)
   finding the scalar product M1
   \[ \cos \angle ABC = 0 \]
   N1
   METHOD 2
29. \( \mathbf{p} \mathbf{w} = p \mathbf{i} + 2p \mathbf{j} - 3p \mathbf{k} \) (seen anywhere) (A1) 

| Attempt to find \( \mathbf{v} + \mathbf{p} \mathbf{w} \) (M1)  
| --- |
| e.g. \( 3\mathbf{i} + 4\mathbf{j} + \mathbf{k} + p(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \)  
| Collecting terms \( (3 + p)\mathbf{i} + (4 + 2p)\mathbf{j} + (1 - 3p) \mathbf{k} \) (A1)  
| Attempt to find the dot product (M1)  
| e.g. \( 1(3 + p) + 2(4 + 2p) - 3(1 - 3p) \)  
| Setting their dot product equal to 0 (M1)  
| e.g. \( 1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0 \)  
| Simplifying  
| e.g. \( 3 + p + 8 + 4p - 3 + 9p = 0, \ 14p + 8 = 0 \)  
| \( P = -0.571 \left( -\frac{8}{14} \right) \) (A1)  
| N3 |

30. (a) (i) evidence of approach (M1)  

| e.g. \( \mathbf{AO} + \mathbf{OB} = \mathbf{AB}, \ \mathbf{B} - \mathbf{A} \)  
| \( \mathbf{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} \) (AG)  
| N0 |

(ii) for choosing correct vectors, \( \mathbf{AO} \) with \( \mathbf{AB} \), or \( \mathbf{OA} \) with \( \mathbf{BA} \) (A1)(A1)  

| Note: Using \( \mathbf{AO} \) with \( \mathbf{BA} \) will lead to \( \pi - 0.799 \). If they then say \( \hat{\mathbf{B}A} \mathbf{O} = 0.799 \), this is a correct solution. |

| Calculating \( \mathbf{AO} \cdot \mathbf{AB}, \ \left| \mathbf{AO} \right| \left| \mathbf{AB} \right| \) (A1)(A1)(A1)  
| e.g. \( d_1 \cdot d_2 = (-1)(-4) + (2)(6) + (-3)(-1) = 19 \)  
| \( \left| d_1 \right| = \sqrt{(-1)^2 + 2^2 + (-3)^2} = \sqrt{14} \)  
| \( \left| d_2 \right| = \sqrt{(-4)^2 + 6^2 + (-1)^2} = \sqrt{53} \)  
| Evidence of using the formula to find the angle (M1)  
| e.g. \( \cos \theta = \frac{(-1)(-4) + (2)(6) + (-3)(-1)}{\sqrt{(-1)^2 + 2^2 + (-3)^2} \sqrt{(-4)^2 + 6^2 + (-1)^2}} \). |
\[\frac{19}{\sqrt{14}} \approx 0.69751\ldots\]

\[\text{BAO} = 0.799\text{ radians (accept 45.8°)}\]  

(a) two correct answers  
\[\text{e.g.} (1, -2, 3), (-3, 4, 2), (-7, 10, 1), (-11, 16, 0)\]  

(b) \[\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}\]  

(ii) C on \(L_2\), so  
\[\begin{pmatrix} k \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}\]  

\[\text{e.g.} 1 - 3t = k, -2 + 4t = -2, 5 = 3 + 2t\]  

one correct value \(t = 1, k = -2\) (seen anywhere)  

coordinates of C are \((-2, 2, 5)\)  

(d) for setting up one (or more) correct equation using  
\[\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}\]  

\[e.g. 3 + p = -2, -8 - 2p = 2, -p = 5\]  

\[p = -5\]  

31. evidence of equating vectors  
\[\text{e.g.} L_1 = L_2\]  

for any two correct equations  
\[\text{e.g.} 2 + s = 3 - t, 5 + 2s = -3 + 3t, 3 + 3s = 8 - 4t\]  

attempting to solve the equations  

finding one correct parameter \((s = -1, t = 2)\)  

the coordinates of T are \((1, 3, 0)\)  

32. (a) (i) evidence of approach  
\[\text{e.g. } \overrightarrow{AO} + \overrightarrow{OB}, \overrightarrow{B} - \overrightarrow{A} = \begin{pmatrix} 9 - 6 \\ -6 + 2 \\ 15 - 10 \end{pmatrix}\]  

\[\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}\]  

(accept (3, 4, 5))  

(ii) evidence of finding the magnitude of the velocity vector  
\[\text{e.g. speed} = \sqrt{3^2 + 4^2 + 5^2}\]
speed = $\sqrt{50} = 5\sqrt{2}$

(b) correct equation (accept Cartesian and parametric forms)

\[
e.g. \quad r = \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}, \quad r = \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}
\]

33. (a) evidence of addition (M1)

e.g. at least two correct elements

\[
A + B = \begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}
\]

(b) evidence of multiplication (M1)

e.g. at least two correct elements

\[
-3A = \begin{pmatrix} -3 & -6 \\ -9 & 3 \end{pmatrix}
\]

(c) evidence of matrix multiplication (in correct order) (M1)

e.g.

\[
AB = \begin{pmatrix} 1(3)+2(-2) & 1(0)+2(1) \\ 3(3)+(-1)(-2) & 3(0)+(-1)(1) \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 11 & -1 \end{pmatrix}
\]

34. (a) \( \det M = -4 \)

(b) \( M^{-1} = -\frac{1}{4} \begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix} \)

Note: Award A1 for \( -\frac{1}{4} \) and A1 for the correct matrix.

(c) \( X = M^{-1} \begin{pmatrix} 4 \\ 8 \end{pmatrix} \)

\( X = -\frac{1}{4} \begin{pmatrix} -1 & -1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \end{pmatrix} \)

\( X = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \) (\( x=3, y=-2 \))

Note: Award no marks for an algebraic solution of the system \( 2x + y = 4, 2x - y = 8 \).

35. (a) evidence of correct method (M1)

e.g. at least 1 correct element (must be in a 2 \times 2 matrix)
\[ AB = \begin{pmatrix} -2 - 2q & 0 \\ -6 + pq & 3 + \frac{p}{2} \end{pmatrix} \]  

(b) **METHOD 1**

evidence of using \( AB = I \) (M1)

2 correct equations A1

e.g. \(-2 - 2q = 1\) and \(3 + \frac{p}{2} = 1, -6 + pq = 0\)

\[ p = -4, q = -\frac{3}{2} \] A1

**METHOD 2**

finding \( A^{-1} = \frac{1}{p + 6} \begin{pmatrix} p & 2 \\ -3 & 1 \end{pmatrix} \) A1

evidence of using \( A^{-1} = B \) (M2)

e.g. \( \frac{2}{p + 6} = 1\) and \(\frac{3}{p + 6} = q, \frac{p}{p + 6} = -2\) and \(\frac{3}{p + 6} = q\)

\[ p = -4, q = -\frac{3}{2} \] A1

---

**36.**

(a) \( A^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & -0.5 & 1.25 \\ 1 & -0.5 & 0.75 \end{pmatrix} \) A2

(b) (i) \( I - \frac{1}{2} B = A^{-1} \) A1

\[ -\frac{1}{2} B = A^{-1} - I \] A1

\[ B = -2(A^{-1} - I) \] AG

(ii) \( B = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 3 & -2.5 \\ -2 & 1 & 0.5 \end{pmatrix} \) A2

(iii) \( \det B = 12 \) A1

(iv) \( \det B \neq 0 \) R1

(c) (i) evidence of using a valid approach M1

e.g. \( X = B^{-1} C \)

\[ X = \begin{pmatrix} 0.333 \\ 1 \\ 1.33 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 1 \\ \frac{4}{3} \end{pmatrix} \] A1

(ii) \( 4x - 2y + 2z = 2, -2x + 3y - 2.5z = -1, -2x + y + 0.5z = 1 \) A1A1A1

N3