SL Practice Problems – Algebra

Name: ____ Date:________________ Block:________

Paper 1 – No Calculator

1. Consider the arithmetic sequence 2, 5, 8, 22, … .
   (a) Find u_{101}. *3 marks*
   (b) Find the value of n so that $u_n = 152$. *3 marks*
2. Consider the infinite geometric sequence 25, 5, 1, 0.2, …
   (a) Find the common ratio.
   (b) Find
      (i) the 10th term;
      (ii) an expression for the nth term.
   (c) Find the sum of the infinite sequence.
3. Given that $p = \log_a 5,\ q = \log_a 2,$, express the following in terms of p and/or q.
   (a) $\log_a 10$
   (b) $\log_a 8$
   (c) $\log_a 2.5$
4. Consider the expansion of the expression $(x^3 - 3x)^6$.
   (a) Write down the number of terms in this expansion.
   (b) Find the term in x^{12}.
5. (a) Given that $(2^x)^2 + (2^x) - 12$ can be written as $(2^x + a)(2^x + b),\ a,b \in \mathbb{Z}$, find the value of a and b.
   (b) hence find the exact value of the equation $(2^x)^2 + (2^x) - 12 = 0$, and explain why there is only one solution.
6. (a) Let $\log_e 3 = p,\ \text{and}\ \log_e 5 = 1.$ Find an expression in terms of p and q for
      (i) $\log_e 15$;
      (ii) $\log_e 25$.
   (b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

Paper 2 –Calculator is Allowed

7. (a) Consider the geometric sequence –3, 6, –12, 24, …
      (i) Write down the common ratio.
      (ii) Find the 15th term. *3 marks*
   Consider the sequence x – 3, x + 1, 2x + 8, …
   (b) When x = 5, the sequence is geometric.
      (i) Write down the first three terms.
      (ii) Find the common ratio. *2 marks*
   (c) Find the other value of x for which the sequence is geometric. *4 marks*
   (d) For this value of x, find
      (i) the common ratio;
      (ii) the sum of the infinite sequence. 3 marks

SL Practice Problems – Functions and Equations

Name: ______________________________________ Date:_______________ Block:________

Paper 1 – No Calculator

1. Let $f(x) = \ln(x+5) + \ln 2$, for $x > -5$.

   (a) Find $f^{-1}(x)$. *4 marks*

   Let $g(x) = e^x$.

   (b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in$ ₵. *3 marks*

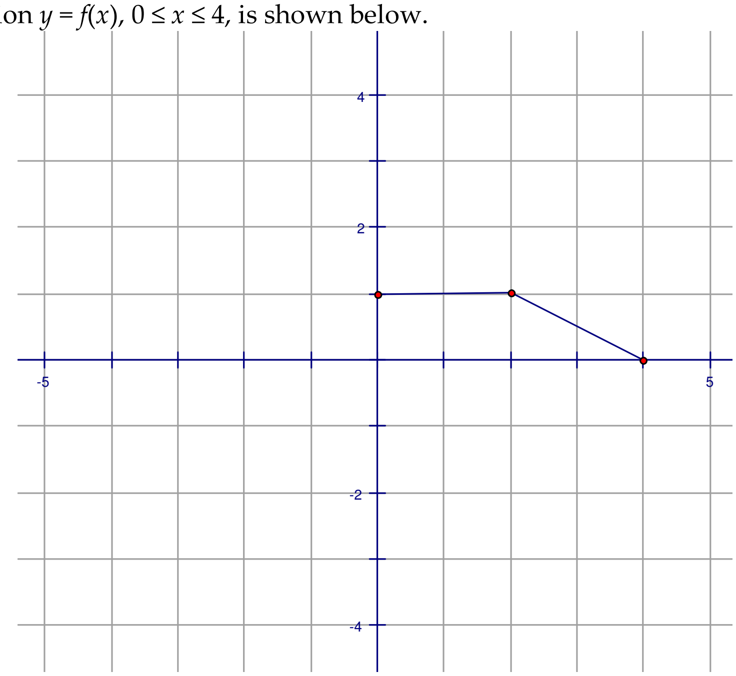
2. The functions f(x) and g(x) are defined by $f(x) = e^x,\ g(x) = \ln(1+2x)$.

   (a) Write down $f'(x)$.

   (b) Find $(f \circ g)(x)$.

   (c) Find $(f \circ g)^{-1}(x)$.

3. The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.



   (a) Write down the value of $f'(1)$ and $f'(3)$.

   (b) On a diagram similar to the one given, draw the graph of $y = 3f(-x)$.

   (c) On a diagram similar to the one given, draw the graph of $y = f(2x)$.

4. Let $f(x) = a(x-4)^2 + 8$.

   (a) Write down the coordinates of the vertex of the curve of f.

   (b) Given that $f(7) = -10$, find the value of a.

   (c) Hence find the y-intercept of the curve of f.

5. Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

   (a) Find $(g \circ f)(-2)$.

   (b) Find $f^{-1}(x)$.

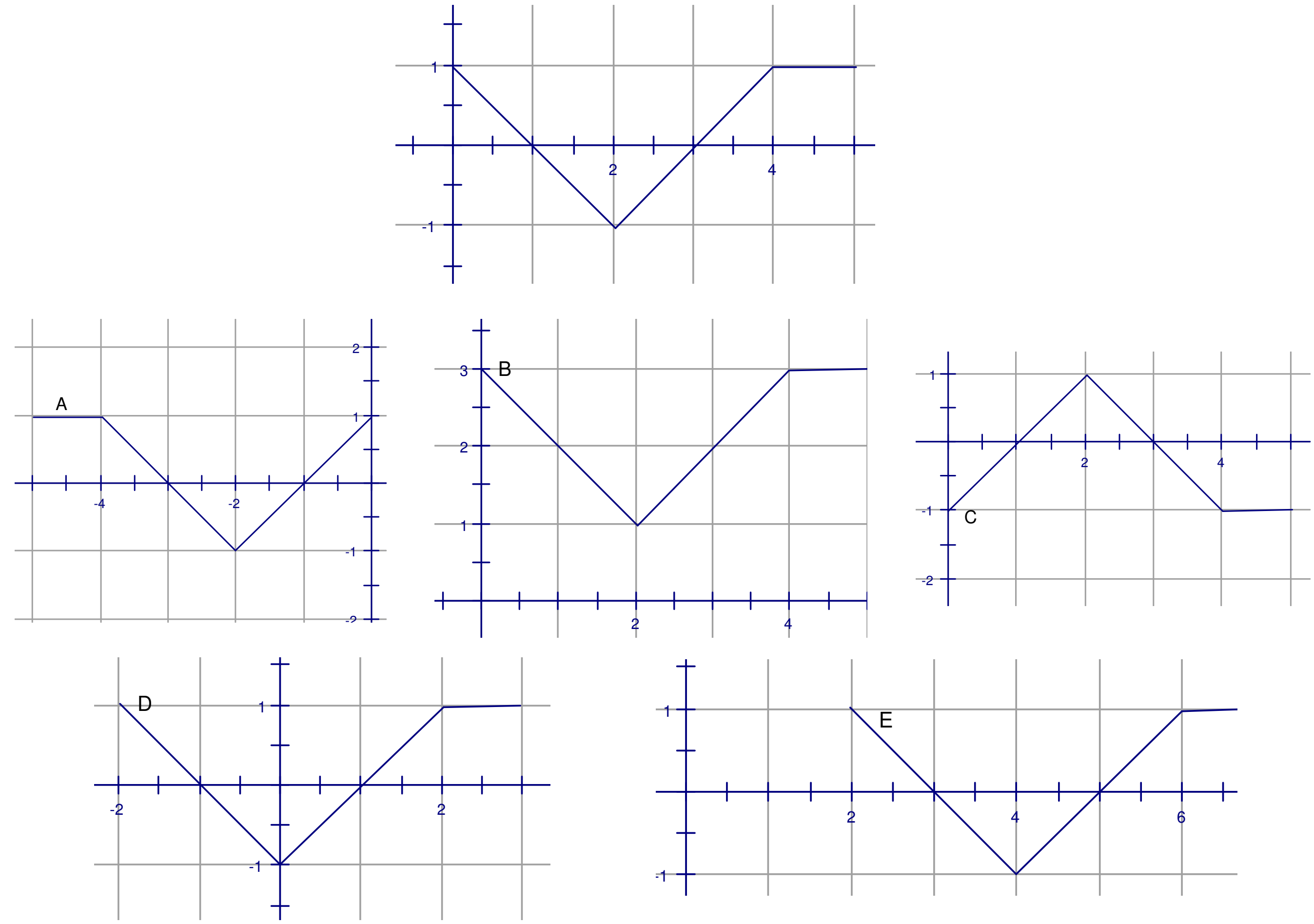
6. Let $f(x) = 3\sin 2x$ for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$.

   (a) On the same diagram, sketch the graph of f and g on the domain given.

   (b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution.

   (c) Let $h(x) = g(x) - f(x)$. Given that $h(x) > 0$ for $p < x < q$, write down the values of p and q.

7. The following diagram shows part of the graph of $f(x)$. Consider the five graphs labeled A, B, C, D, E below it.



(a) Which diagram is the graph of $f(x+2)$?

(b) Which diagram is the graph of $-f(x)$?

(c) Which diagram is the graph of $f(-x)$?

Paper 2 –Calculator is Allowed

8. Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. *2 marks*

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts. *8 marks*

(c) **Hence** sketch the graph of f. *2 marks*

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

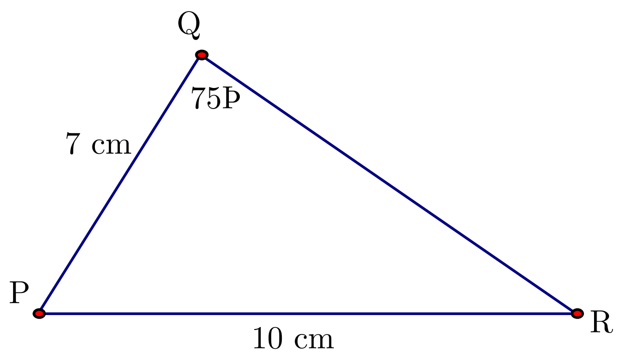
a stretch of scale factor t in the y-direction followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$. Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. *3 marks*

SL Practice Problems – Circular Functions and Trig

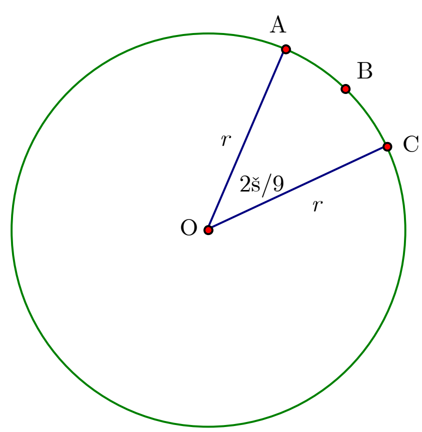
Name: ____ Date:_________________ Block:_________

Paper 1 – No Calculator

1. Consider $g(x) = 3\sin(2x)$.
   - (a) Write down the period of g. *1 mark*
   - (b) Sketch the curve of g, for $0 \le x \le 2\pi$. (Grid given.) *3 marks*
   - (c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$. *2 marks*
2. Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q.
   - (a) Write down an expression for $\sin 140^\circ$ and $\cos 70^\circ$.f *2 marks*
   - (b) Find an expression for $\cos 140^\circ$. *3 marks*
   - (c) Find an expression for $\tan 140^\circ$. *1 mark*
3. The diagram below shows ΔPQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and the $m\angle\mathrm{PQR}$ is 75°.

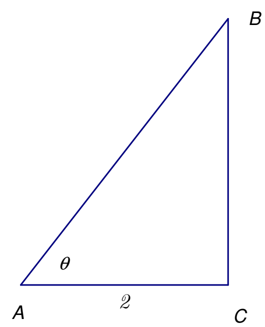


   - (a) Find $m\angle\mathrm{PQR}$. *3 marks*
   - (b) Find the area of ΔPQR. *3 marks*
4. The diagram below shows a circle with center O and radius r. The length of arc ABC is 3π cm and $m\angle\mathrm{AOC} = 2\pi/9$.



   - (a) Find the value of r. *2 marks*
   - (b) Find the perimeter of sector OABC. *2 marks*
   - (c) Find the area of sector OABC. *2 marks*
5. In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm. Calculate
   - (a) the size of $\measuredangle PQR$;
   - (b) the area of triangle PQR.

6. The following diagram shows a triangle *ABC*, where $\measuredangle ACB$ is 90°, *AB* = 3, *AC* = 2 and $\measuredangle BAC = \theta$.

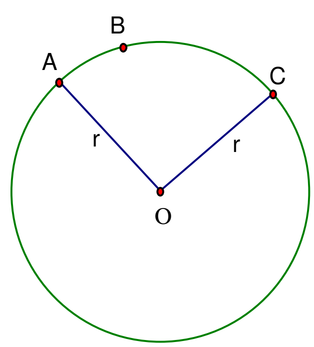


(a) Show that $\sin\theta = \dfrac{\sqrt{5}}{3}$.

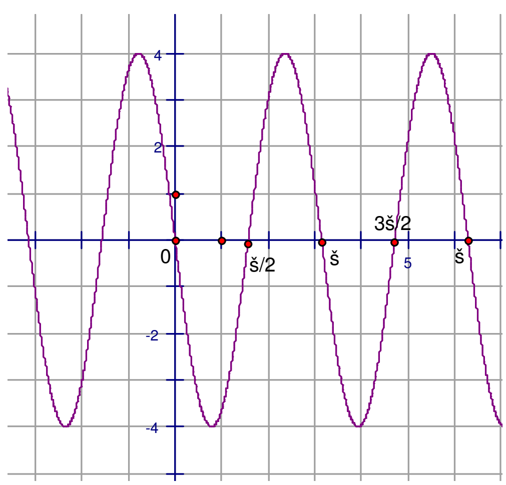
(b) Show that $\sin 2\theta = \dfrac{4\sqrt{5}}{9}$.

(c) Find the exact value of $\cos 2\theta$.

7. The following diagram shows a circle with radius r and center O. The points A, B and C are on the circle and $A\hat{O}C = \theta$. The area of the section OABC is $\frac{4}{3}\pi$ and the length is $\frac{2}{3}\pi$. Find the value of *r* and of θ.



8. Let $f(x) = a\sin b(x-c)$. Part of the graph of *f* is given below. Given that *a*, *b* and *c* are positive, find their values.



9. Let $f(x) = 3\sin 2x$ for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$.

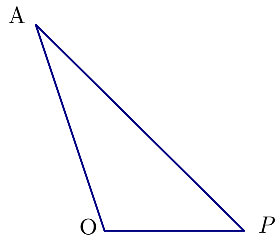
(a) On the same diagram, sketch the graph of *f* and *g* on the domain given.

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution.

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) > 0$ for $p < x < q$, write down the values of *p* and *q*.

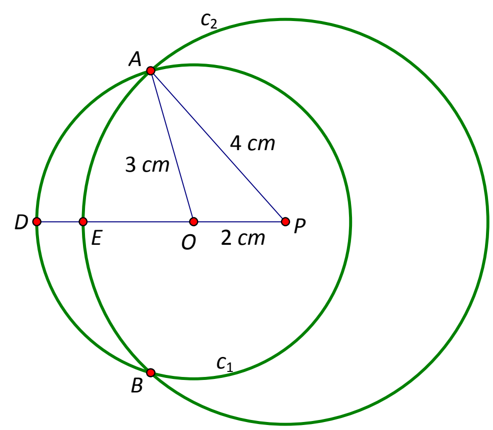
Paper 2 –Calculator is allowed

10. The following diagram shows the triangle *AOP*, where *OP* = 2 cm, *AP* = 4 cm and *AO* = 3 cm. *The diagram is not to scale.*



(a) Calculate $\measuredangle AOP$, giving your answer in radians. *3 marks*

The diagram to the right shows two circles which intersect at the points *A* and *B*. The smaller circle C_1 has center *O* and radius 3 cm, the larger circle C_2 has center *P* and radius 4 cm, and *OP* = 2 cm. The point *D* lies on the circumference of C_1. Triangle AOP is the same triangle as in the diagram above. *The diagram is not drawn to scale.*



(b) Find $\measuredangle AOB$, giving your answer in **radians**. *2 marks*

(c) Given that $\measuredangle APB$ is 1.63 radians, calculate the area of

(i) sector *PAEB*;

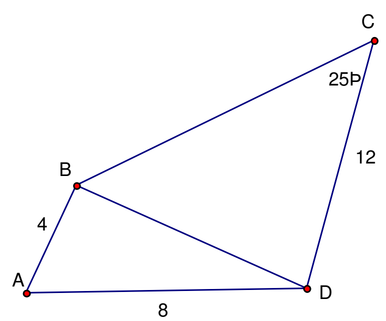
(ii) sector *OADB*. *5 marks*

(d) The area of the quadrilateral *AOBP* is 5.81 cm².

(i) Find the area of *AOBE*.

(ii) Hence find the area of the shaded region *AEBD*. *4 marks*

11. The diagram below shows a quadrilateral *ABCD*, with *AB* = 4, *AD* = 8, *CD* = 12, $B\hat{C}D = 25Þ$, $B\hat{A}D = \theta$.



(a) Use the cosine rule to show that $BD = 4\sqrt{5 - 4\cos\theta}$. *2 marks*

Let $\theta = 40Þ$.

(b) (i) Find the value of $\sin C\hat{B}D$.

(ii) Find the two possible values for the size of $C\hat{B}D$.

(iii) Given that $C\hat{B}D$ is an acute angle, find the perimeter of ABCD. *12 marks*

(c) Find the area of triangle *ABD*. *2 marks*

SL Practice Problems – Matrices

Name: ____ Date:________________ Block:________

Paper 1 – No Calculator

1. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$. Find

   (a) $A + B$; *2 marks*
   (b) $-3A$; *2 marks*
   (c) AB. *3 marks*

2. (a) Write down the inverse of the matrix $A = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix}$.

   (b) Hence or otherwise solve $\begin{cases} x - 3y = 1 \\ 2x + z = 2 \\ 4x + y + 3z = -1 \end{cases}$

3. (a) Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

   (i)Write down the value of *a*
   (ii)Find the value of *b*.

   (b) Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ -9 & 23 \end{pmatrix}$. Find the value of *q*.

4. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

   (a) Write down the inverse matrix A^{-1}.
   (b) Consider the equation $AX = B$.

   (i)Epxress *X* in terms of A^{-1} and *B*.
   (ii)**Hence,** solve for *X*.

Paper 2 –Calculator is allowed

(Need to find some problems for these)

SL Practice Problems – Vectors

Name: ____ Date:________________ Block:________

Paper 1 – No Calculator

1. Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. The vector $\mathbf{v} + p\mathbf{w}$ is perpendicular to **w**. Find the value of *p*. *7 marks*

Paper 2 –Calculator is allowed

2. The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

   (a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

   (ii) Find $m\angle BAO$. *7 marks*

   (b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$. Write down the coordinates of two points on this line. *2 marks*

   (c) The line L_2 passes through A and is parallel to $\overrightarrow{OB}$.

   (i) Find a vector equation for L_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

   (ii) Point C(k, $-k$, 5) is on L_2. Find the coordinates of C. *7 marks*

   (d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through C. Find the value of p at C.

   2 marks

3. Points P and Q have position vectors $-5\vec{i} + 11\vec{j} - 8\vec{k}$ and $-4\vec{i} + 9\vec{j} - 5\vec{k}$ respectively, and both lie on a line L_1.

   (a) (i) Find $\overrightarrow{PQ}$.

   (ii) Hence show that the equation of L_1 can be written as

   $$\vec{r} = (-5 + s)\vec{i} + (11 - 2s)\vec{j} + (-8 + 3s)\vec{k}.$$

   The point $R(2, y_1, z_1)$ also lies on L_1.

   (b) Find the value of y_1 and of z_1. *4 marks*

   The line L_2 has equation $\vec{r} = 2\vec{i} + 9\vec{j} + 13\vec{k} + t(\vec{i} + 2\vec{j} + 3\vec{k})$.

   (c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T. *7 marks*

   (d) Calculate the angle between the lines L_1 and L_2. *7 marks*

4. The position vector of point A is $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and the position of point B is $4\mathbf{i}-5\mathbf{j}+21\mathbf{k}$.

(a) (i) Show that $\mathbf{AB}=2\mathbf{i}-8\mathbf{j}+20\mathbf{k}$.

(ii) Find the unit vector **u** in the direction of **AB**.

(iii) Show that **u** is perpendicular to **OA**. *6 marks*

Let S be the midpoint of [AB]. Line L_1 contains S and is parallel to **OA.**

(b) (i) Find the position vector of S.

(ii) Write down the equation of L_1. *4 marks*

The line L_2 has equation $\mathbf{r}=(5\mathbf{i}+10\mathbf{j}+10\mathbf{k})+s(-2\mathbf{i}+5\mathbf{j}-3\mathbf{k})$.

(c) Explain why L_1 and L_2 are not parallel. *2 marks*

(d) The lines L_1 and L_2 intersect at P. Find the position vector of P. *7 marks*

SL Practice Problems – Statistics and Probability

Name: ____ Date:_______________ Block:________

Paper 1 – No Calculator

1. A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.
   (a) Find the mean number of defective switches in the sample. *2 marks*
   (b) Find the probability that there are exactly six defective switches in the sample. *2 marks*
   (c) Find the probability that there is at least one defective switch in the sample. *3 marks*

2. In a school with 125 girls, each student is tested to see how many sit-ups she can do in one minute. The results are given in the table below.

Number of sit-ups	Number of students	Cumulative number of students
15	11	11
16	21	32
17	33	p
18	q	99
19	18	117
20	8	125

   (a) (i) Write down the value of p.
   (ii) Find the value of q. *3 marks*
   (b) Find the median numbers of sit-ups. *2 marks*
   (c) Find the mean number of sit-ups. *2 marks*

3. Consider the events *A* and *B* where $P(A)=\frac{2}{5},\ P(B')=\frac{1}{4},$ and $P(A\cup B)=\frac{7}{8}$.
   (a) Write down $P(B)$.
   (b) Find $P(A\cap B)$.
   (c) Find $P(A/B)$.

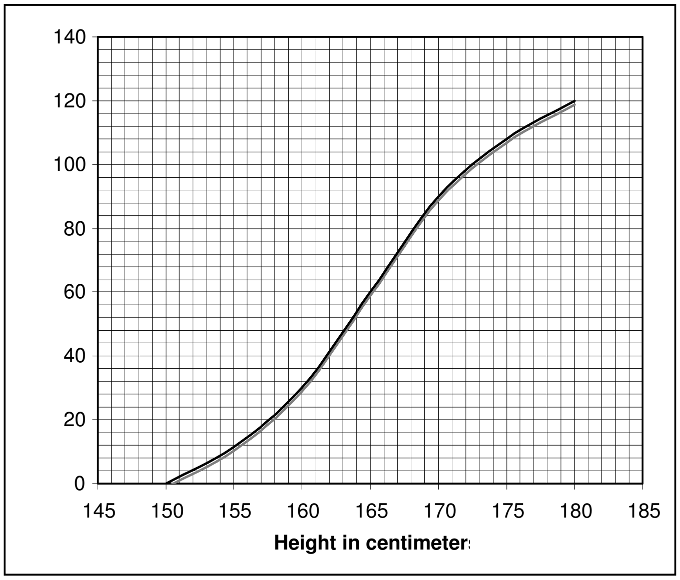
4. The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.
   (a) Calculate the mean height of the boys.
   (b) Find the probability of a boy being taller than 156 cm.

5. Let *A* and *B* be independent events such that $P(A)=0.3,\ P(B)=0.8$.
   (a) Find $P(A\cap B)$.
   (b) Find $P(A\cup B)$.
   (c) Are *A* and *B* mutually exclusive? Justify your answer.

6. Consider the four numbers a, b, c, d with $a \le b \le c \le d$ where $a,b,c,d \in \mathbb{Z}$. The mean of the four numbers is 4, the mode is 3, the median is 3 and the range is 6. Find the value of a, b, c, and d.

7. The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.
   (a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm.
   (b) In this group of students, 11.9% have heights less than d cm. Find the value of d.

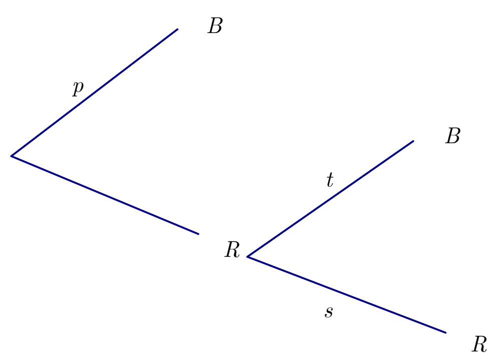
8. The cumulative frequency graph below shows the heights of 120 girls in a school.



(a) Using the graph
  (i) write down the median;
  (ii) find the interquartile range.
(b) Given that 60% of the girls are taller than *a* cm, find the value of *a*.

Paper 2 –Calculator is allowed

9. A **four-sided** die has three blue and one red face. The die is rolled. Let *B* be the event a blue face lands down, and *R* be the event a red face lands down.
(a) Write down P(*B*) and P(*R*). *2 marks*
(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where *p*, *s*, *t* are probabilities.



Find the values of *p*, *s*, and *t*. *2 marks*

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let *X* be the total score obtained.
(c) (i) Show that P(X = 3) = 3/16.
  (ii) Find P(X = 2). *3 marks*
(d) (i) Construct a probability distribution table for *X*.
  (ii) Calculate the expected value of *X*. *5 marks*
(e) If the total score is 3, Guiseppi wins $10. If the total score is 2, Guiseppi gets nothing. Guiseppi plays the game twice. Find the probability that he wins exactly $10.
4 marks

10. A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

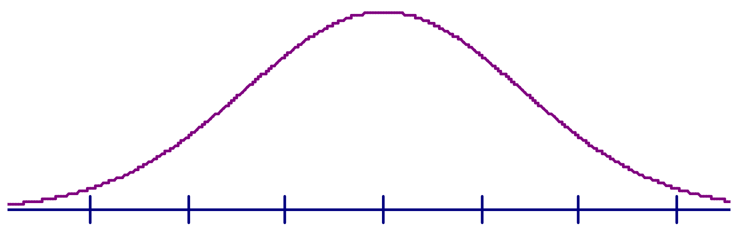
(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g;

(ii) weighs between 6g and 8g. *4 marks*

(b) Five percent of the biscuits in the box weight less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



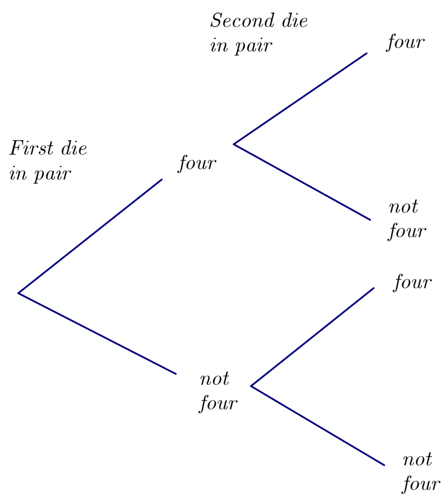
(ii) Find the value of d. *5 marks*

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weight less than 5 g. Find the value of μ.

4 marks

11. A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



Let E be the event that exactly one four occurs when the pair of dice is thrown.

(b) Calculate $P(E)$. *3 marks*

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws. *3 marks*

(d) Calculate the probability that event E occurs **at least** three times in the five throws. *3 marks*

12. **Part A**. Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4 or 9 written on it.

(a) Kim states that the probability distribution for her pack of cards is as follows. Explain why she is wrong. *2 marks*

x	0	3	4	9
$P(X = x)$	0.3	0.45	0.2	0.35

(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows. Find the value of k. *2 marks*

x	0	3	4	9
$P(X = x)$	0.4	k	$2k$	0.3

(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $P(x) = \dfrac{x+1}{20}$. One card is drawn from his pack.

(i) Calculate the probability that the number on the card is 0.

(ii) Calculate the probability that the number on the card drawn is greater than 0. *4 marks*

Part B. A game is played, where a die is tossed and a marble selected from a bag. Bag M contained 3 red marbles (R) and 2 green marbles (G). Bag N contains 2 red marbles and 8 green marbles. A fair six-sided die is tossed. If a 3 or 5 appears on the die, bag M is selected (M). If any other number appears, bag N is selected (N). A single marble is then drawn from the selected bag.

(a) Complete the following probability tree diagram.

(b) (i) Write down the probability that bag M is selected and a green marble drawn from it.

(ii) Find the probability that a green marble is drawn from either bag.

(iii) Given that the marble is green, calculate the probability that it came from bag M. *7 marks*

(c) A player wins \$2 for a red ball and \$5 for a green ball. What are his expected winnings? *4 marks*

SL Practice Problems – Calculus

Name: ____ Date:_________________ Block:_________

Paper 1 – No Calculator

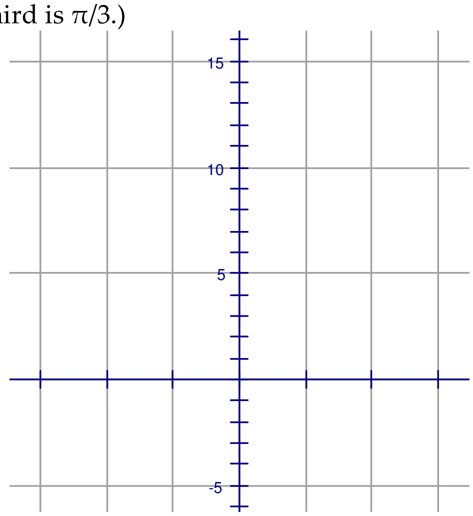
1. A particle moves along a straight line so that its velocity, v ms^{-1} at time *t* seconds is given by $v = 6e^{3t} + 4$. When *t* = 0, the displacement, *s*, of the particle is 7 meters. Find an expression for *s* in terms of *t*. *7 marks*

2. (a) Find $\int \frac{1}{2x+3}\,dx$. *2 marks*

   (b) Given that $\int_0^3 \frac{1}{2x+3}\,dx = \ln\sqrt{P}$, find P. *4 marks*

3. Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \le x \ge \frac{\pi}{3}$.

   (a) On the grid below, sketch the graph of $y = f(x)$. (The markings on the *x*-axis are each incremented by π/9, so the third is π/3.) *3 marks*



   (b) Solve the equation $f(x) = 1$. *3 marks*

4. The velocity *v* of a particle at time *t* is given by $v = e^{-2t} + 12t$. The displacement of the particle at time *t* is *s*. Given that *s* = 2 when *t* = 0, express *s* in terms of *t*.

5. Let $f(x) = x^3 - 3x^2 - 24x + 1$. The tangents to the curve of *f* at the points *P* and *Q* are parallel to the *x*-axis, where *P* is to the left of *Q*.

   (a) Calculate the coordinates of *P* and of *Q*.

   Let *N* and *M* be the normals to the curve at *P* and *Q*, respectively.

   (b) Write down the coordinates of the points where the tangent at *P* meets *M* and the tangent at *Q* meets *N*.

6. It is given that $\int_1^3 f(x)dx = 5$.

   (a) Write down the value of $\int_1^3 2f(x)dx$.

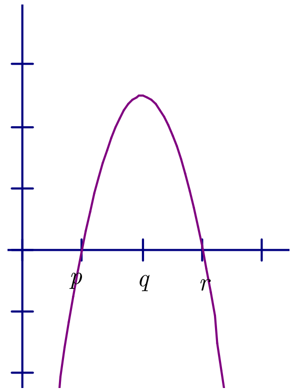
   (b) Find the value of $\int_1^3 (3x^2 + f(x))dx$.

7. (a) Let $f(x) = e^{5x}$. Write down $f'(x)$.

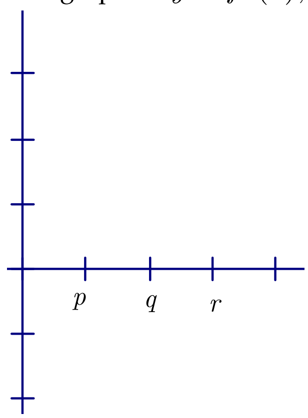
   (b) Let $g(x) = \sin 2x$. Write down $g'(x)$.

   (c) Let $h(x) = e^{5x}\sin 2x$. Find $h'(x)$.

8. The diagram below shows part of the graph of the **gradient (slope)** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept. *2 marks*

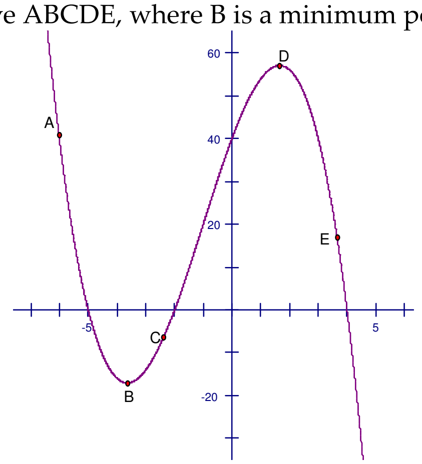


(b) Complete the table for the graph of $y = f(x)$. *2 marks*

	x-coordinate
(i) Maximum point on f	
(ii) Inflection point on f	

(c) Justify your answer to part (b) (ii). *2 marks*

9. The following diagram shows a curve ABCDE, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	B	E
$f''(x)$			

10. The velocity, v m/s, of a moving object at time t seconds is given by $v = 4t^3 - 2t$. When $t = 2$, the displacement, s, of the object is 8 meters. Find an expression for s in terms of t.

Paper 2 –Calculator is allowed

11. The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

(a) Sketch the curve of f for $-5 \le x \le 5$, showing the asymptotes. *3 marks*

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

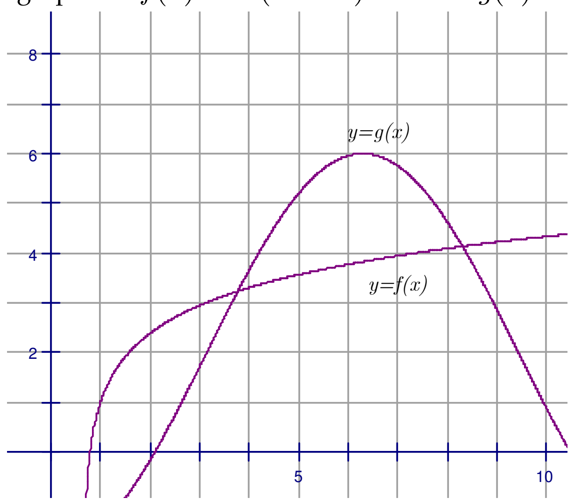
(iii) the value of the y-intercept. *4 marks*

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

(i) Find $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx$.

(ii) Hence, given that $V = \pi\left(\frac{28}{3} + 3\ln 3\right)$, find the value of a. *10 marks*

12. The diagram shows graphs of $f(x) = \ln(3x-2)+1$ and $g(x) = -4\cos(0.5x)+2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves f and g.

(i) Find an expression for A.

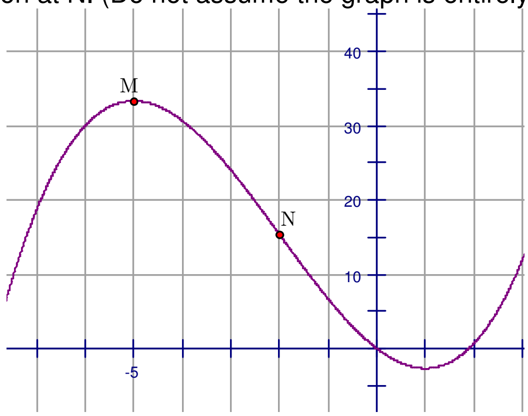
(ii) Calculate the value of A. *6 marks*

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$. *4 marks*

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. *4 marks*

13. Consider $f(x)=\dfrac{1}{3}x^3+2x^2-5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflection at N. (Do not assume the graph is entirely correct.)



(a) Find $f'(x)$. *3 marks*

(b) Find the x-coordinate of M. *4 marks*

(c) Find the x-coordinate of N. *3 marks*

(d) The line L is the tangent to the curve of f at (3, 12). Find the equation of L in the form $y = mx + b$. *4 marks*

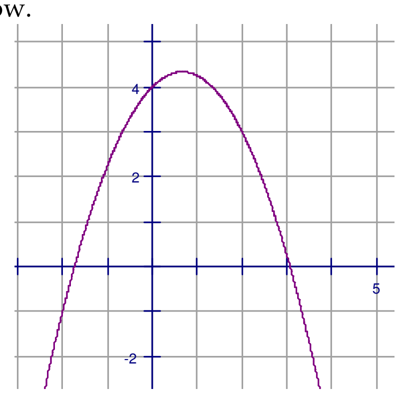
14. Let $f(x)=-\frac{3}{4}x^2+x+4$.

(a) (i) Write down $f'(x)$.

(ii) Find the equation of the normal to the curve of f at (2, 3).

(iii) This normal intersects the curve of f again at a point P. Find the x-coordinately of P. *9 marks*

Part of the graph of f is given below.



(b) Let R be the region under the curve of f from $x = -1$ to $x = 2$.

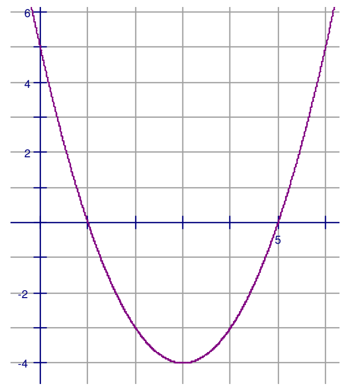
(i) Write down an expression for the area of R.

(ii) Calculate this area.

(iii) The region R is rotated through 360˚ about the x-axis. Write down an expression for the volume of the solid formed. *6 marks*

(c) Find $\int_{-1}^{k} f(x)\ dx$, giving your answer in terms of k. *6 marks*

15. **Part A.** The following graph shows part of the graph of a quadratic function, with equation in the form $y=(x-p)(x-q),\ p,q \in$ ¢ .



(a) (i) Write down the values of p and q.

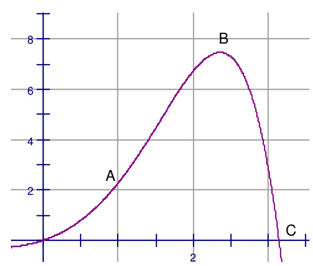
(ii) Write down the equation of the axis of symmetry of the curve. *3 marks*

(b) Find the equation of the function in the form $y=(x-h)^2+k$, where $h,k \in \mathbb{Z}$. *3 marks*

(c) Find $\dfrac{dy}{dx}$. *2 marks*

(d) Let T be the tangent to the curve at the point (0, 5). Find the equation of T. *2 marks*

Part B. The function f is defined as $f(x)=e^x \sin x$, where x is in radians. Part of the curve of f is shown below. There is a point of inflection at A, and a local maximum at B. The curve of f intersects the x-axis at point C.



(a) Write down the x-coordinate of the point C.

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B. *4 marks*

(c) Show that $f''(x)=2e^x \cos x$. *2 marks*

(d) (i) Write down the value of $f''(x)$ at A, the point of inflection.

(ii) Hence, calculate the coordinates of A. *4 marks*

(e) Let R be the region enclosed by the curve and the x-axis, between the origin and C.

(i) Write down an expression for the area of R.

(ii) Find the area of R. *4 marks*