Pre-DP Final Exam Practice Problems

The final will cover the 4 main topics that we studied this year 1) Vectors and Parametric Equations, 2) Circles and their equations, 3) Quadratic functions, and 4) Probability and Statistics.

Note: In addition to working these problems, you should review all your quizzes, tests, and your Semester Final.

1. A line L passes through A(1, –1, 2) and is parallel to the line $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(2)

The line L passes through point P when $t = 2$.

(b) Find

(i) $\overrightarrow{OP}$;

(ii) $\left|\overrightarrow{OP}\right|$.

(4)

(Total 6 marks)

2. In this question, a unit vector represents a displacement of 1 metre. A miniature car moves in a straight line, starting at the point (2, 0). After t seconds, its position, (x, y), is given by the vector equation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0.7 \\ 1 \end{pmatrix}$$

(a) How far from the point (0, 0) is the car after 2 seconds?

(2)

(b) Find the speed of the car.

(2)

(c) Obtain the equation of the car's path in the form $ax + by = c$.

(2)

Another miniature vehicle, a motorcycle, starts at the point (0, 2), and travels in a straight line with constant speed. The equation of its path is

$$y = 0.6x + 2, \quad x \geq 0.$$

Eventually, the two miniature vehicles collide.

(d) Find the coordinates of the collision point.

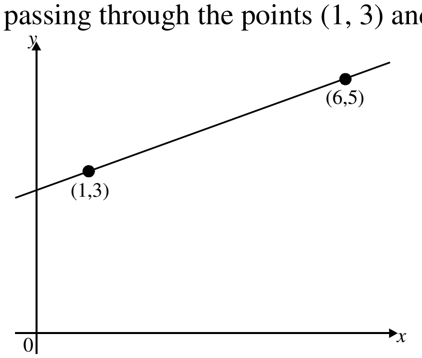
(3)

(e) If the motorcycle left point (0, 2) at the same moment the car left point (2, 0), find the speed of the motorcycle.

(5)

(Total 14 marks)

3. The diagram below shows a line passing through the points (1, 3) and (6, 5).



Find a vector equation for the line, giving your answer in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + t\begin{pmatrix} c \\ d \end{pmatrix}, \text{ where } t \text{ is any real number.}$$

(Total 4 marks)

4. The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are perpendicular for two values of x.

(a) Write down the quadratic equation which the two values of x must satisfy.

(b) Find the two values of x.

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

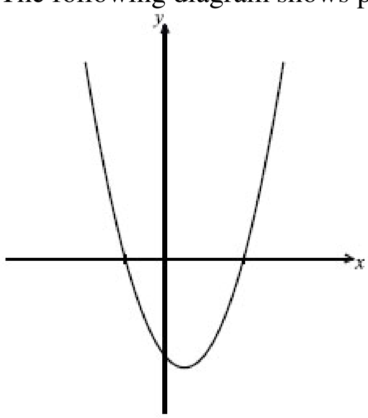
5. The line L passes through the origin and is parallel to the vector $2\boldsymbol{i} + 3\boldsymbol{j}$.
Write down a vector equation for L.

Working:

Answer:
....

(Total 4 marks)

6. The following diagram shows part of the graph of f, where $f(x) = x^2 - x - 2$.



(a) Find both x-intercepts. **(4)**

(b) Find the x-coordinate of the vertex. **(2)**

(Total 6 marks)

7. Let $f(x) = 2x^2 + 4x - 6$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$. **(3)**

(b) Write down the equation of the axis of symmetry of the graph of f. **(1)**

(c) Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$. **(2)**

(Total 6 marks)

8. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

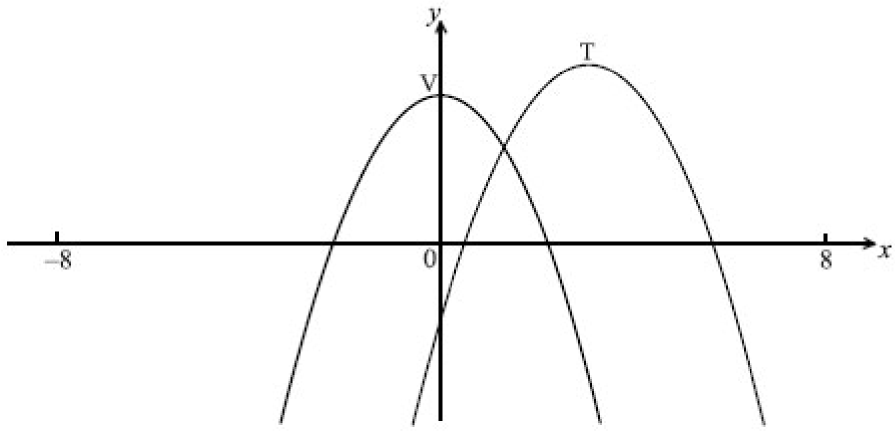
(a) Write f in the form $f(x) = 3(x - h)^2 - k$. **(3)**

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$. **(3)**

(Total 6 marks)

9. The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V (0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T (3, 6).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Write down an expression for $g(x)$.

(2)

(c) On the same diagram, sketch the graph of $y = g(-x)$.

(2)

(Total 6 marks)

In addition to understanding these problems, be sure to review circles and probability/statistics.

Pre-DP Final Exam Practice Problems - MarkScheme

1. (a) correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ A2 N22

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

(b) (i) attempt to substitute $t = 2$ into the equation (M1)

e.g. $\begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$ 3 A1 N2

(ii) correct substitution into formula for magnitude A1

e.g. $\sqrt{3^2 + 5^2 + (-2)^2}, \sqrt{3^2 + 5^2 + 2^2}$

$\left|\overrightarrow{OP}\right| = \sqrt{38}$ A1 N14

[6]

2. (a) At $t = 2$, $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0.7 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.4 \\ 2 \end{pmatrix}$ (M1)

Distance from (0, 0) = $\sqrt{3.4^2 + 2^2}$ = 3.94 m (A1) 2

(b) $\left|\begin{pmatrix} 0.7 \\ 1 \end{pmatrix}\right| = \sqrt{0.7^2 + 1^2}$ (M1)

= 1.22 m s^{-1} (A1) 2

(c) $x = 2 + 0.7\,t$ and $y = t$ (M1)

$x - 0.7y = 2$ (A1) 2

(d) $y = 0.6x + 2$ and $x - 0.7y = 2$ (M1)

$x = 5.86$ and $y = 5.52$ $\left(\text{or } x = \frac{170}{29} \text{ and } y = \frac{160}{29}\right)$ (A1)(A1)3

(e) The time of the collision may be found by solving

$\begin{pmatrix} 5.86 \\ 5.52 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.7 \\ 1 \end{pmatrix} t$ for t (M1)

$\Rightarrow t = 5.52$ s (A1)

[*ie* collision occurred 5.52 seconds after the vehicles set out].

Distance d travelled by the motorcycle is given by

$d = \left|\begin{pmatrix} 5.86 \\ 5.52 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right| = \sqrt{(5.86)^2 + (3.52)^2}$ (M1)

$= \sqrt{46.73}$

= 6.84 m (A1)

Speed of the motorcycle = $\frac{d}{t} = \frac{6.84}{5.52}$

= 1.24 m s^{-1} (A1) 5

[14]

3. Direction vector = $\begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ (A1)

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ (A2)

OR

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} + t\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ (A2) (C4)

[4]

4. (a) $\begin{pmatrix} 2x \\ x-3 \end{pmatrix} \bullet \begin{pmatrix} x+1 \\ 5 \end{pmatrix} = 0$ (M1)(M1)

$\Rightarrow 2x(x + 1) + (x - 3)(5) = 0$ (A1)

$\Rightarrow 2x^2 + 7x - 15 = 0$ (C3)

(b) **METHOD 1**

$2x^2 + 7x - 15 = (2x - 3)(x + 5) = 0$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

METHOD 2

$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

[4]

5. Vector equation of a line $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{t}$ (M1)

$\boldsymbol{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{t} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (M1)(M1)

$\Rightarrow \boldsymbol{r} = \lambda(2\boldsymbol{i} + 3\boldsymbol{j})$ (A1)

(C4)

[4]

6. (a) evidence of attempting to solve $f(x) = 0$ (M1)

evidence of correct working A1

e.g. $(x+1)(x-2), \frac{1 \pm \sqrt{9}}{2}$

intercepts are (−1, 0) and (2, 0) (accept $x = -1$, $x = 2$) A1A1N1N1

(b) evidence of appropriate method (M1)

e.g. $x_v = \frac{x_1 + x_2}{2}$, $x_v = -\frac{b}{2a}$, reference to symmetry

$x_v = 0.5$ A1 N2

[6]

7. (a) evidence of obtaining the vertex (M1)

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

$f(x) = 2(x + 1)^2 - 8$ A2 N3

(b) $x = -1$ (equation must be seen) A1 N1

(c) $f(x) = 2(x - 1)(x + 3)$ A1A1 N2

[6]

intercepts are (−1, 0) and (2, 0) (accept $x = -1$, $x = 2$) A1A1N1N1

(b) evidence of appropriate method (M1)

e.g. $x_v = \frac{x_1 + x_2}{2}$, $x_v = -\frac{b}{2a}$,

8. (a) For a reasonable attempt to complete the square, (or expanding) (M1)

e.g. $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$f(x) = 3(x - 2)^2 - 1$ (accept $h = 2$, $k = 1$) A1A1 N3

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

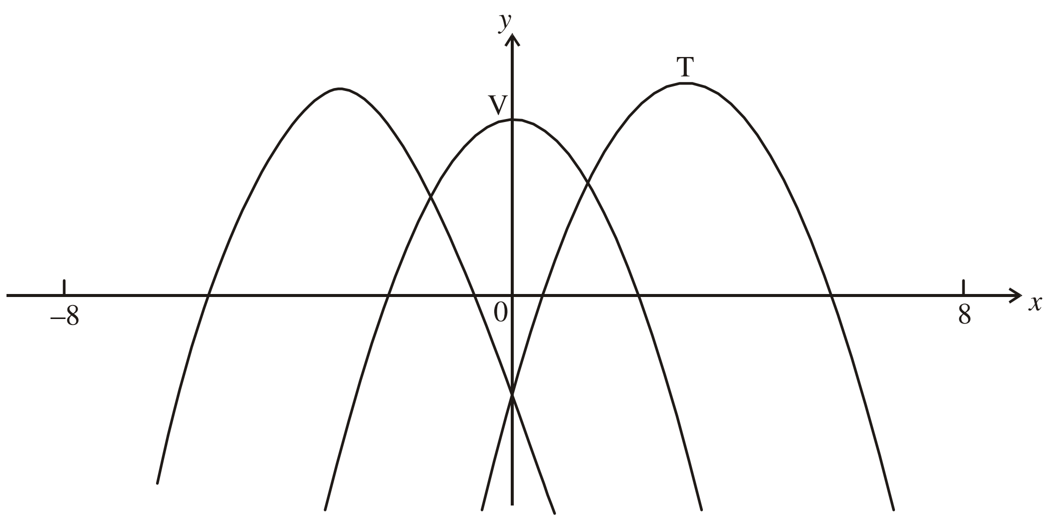
[6]

9. (a) (i) $h = 3$ A1 N1

(ii) $k = 1$ A1 N1

(b) $g(x) = f(x - 3) + 1$, $5 - (x - 3)^2 + 1$, $6 - (x - 3)^2$, $-x^2 + 6x - 3$ A2 N2

(c)



M1A1 N2

Note: *Award M1 for attempt to reflect through y-axis, A1 for vertex at approximately (−3, 6).*

[6]