Fishing Rods

A Type II Portfolio by:

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Fishing Rods

In this portfolio I will be building a model for the relationship of guides on a fishing rod and their distance from the tip of the fishing rod.

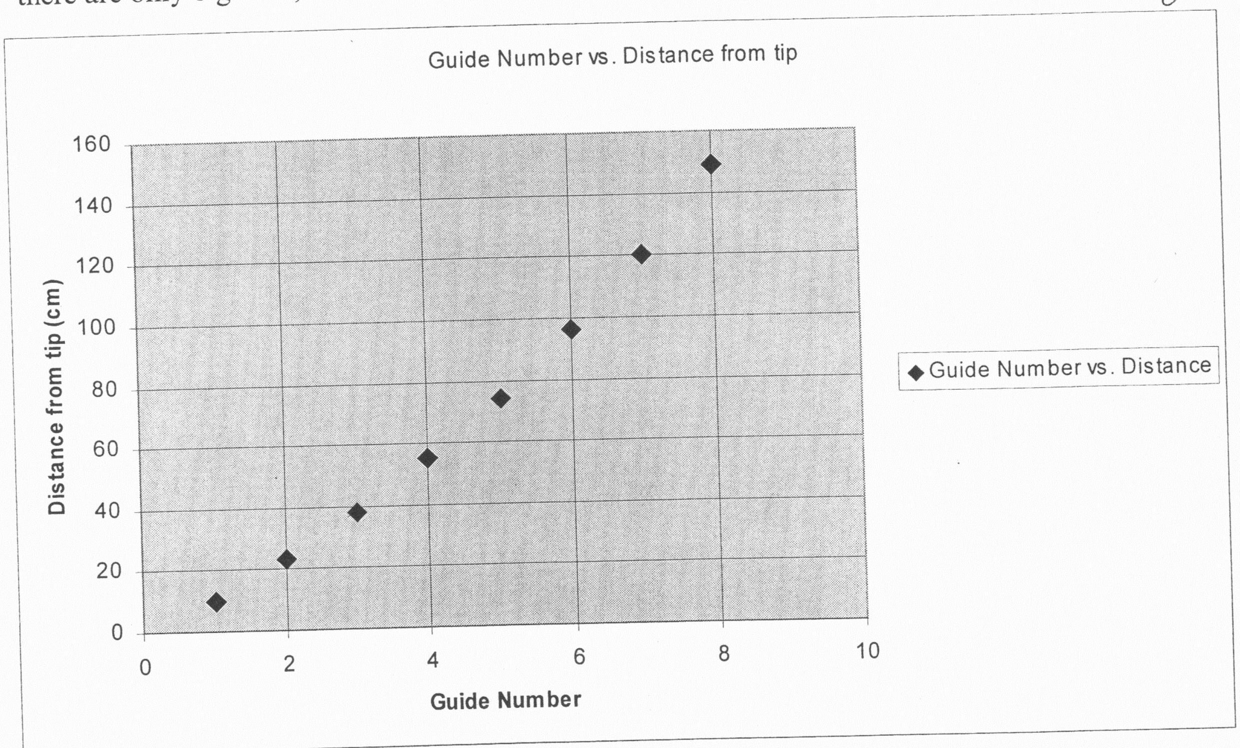
Nice!

The table below gives the distance from the tip of a fishing rod to each guide on a 230 cm. long fishing rod.

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10.00	23.00	38.00	55.00	74.00	96.00	120.00	149.00

In this graph I will let distance = Y and Guide number = X. X must range from *1* to *8* as there are only 8 guides, and Y must range from *10* to *149*.

Variable ~~parameters~~ constraints Good Job!



These points look as though they are a quadratic. I can test this by taking the values and trying to find a pattern.

When X=1, Y=10
When X=2, Y=23
When X=3, Y=38
When X=4, Y=55
When X=5, Y=74

Here one can see a pattern in the Y values, for each step the Y values increase by a set amount. The first step, from 1 to 2, is 13. The next, from 2 to 3, is 15. After that, from 3 to 4, the step is 17. Here I see a constant increase of the step before it plus 2.

I will prove this by finding a quadratic that fits the data. In order to do this I will use substitution in figuring out a system of equations.

Assuming that this data creates a quadratic, I will use the equation $Y = aX^2 + bX + c$.

In setting up a system of equations I will use the three points; (1,10),(2,23),(3,38), so I get the equations:

$$a(1)^2 + b(1) + c = 10$$
$$a(2)^2 + b(2) + c = 23$$
$$a(3)^2 + b(3) + c = 38$$

Or

$$a + b + c = 10$$
$$4a + 2b + c = 23$$
$$9a + 3b + c = 38$$

In order to solve this system of equations I will first solve for on variable, that being *c*. I will do this using elimination.

$$\begin{array}{r} 4a + 2b + c = 23 \\ \underline{a + b + c = 10} \end{array} \quad \text{and} \quad \begin{array}{r} 9a + 3b + c = 38 \\ \underline{4a + 2b + c = 23} \end{array}$$

(Done using GDC)

In order to make this work, I will make the whole second equation negative to eliminate the *c*. I will **bold** the changes in order to make them clearer.

$$\begin{array}{r} 4a + 2b + c = 23 \\ \underline{\mathbf{-a} + \mathbf{-b} + \mathbf{-c} = \mathbf{-10}} \\ 3a + b \quad = 13 \end{array} \quad \text{and} \quad \begin{array}{r} 9a + 3b + c = 38 \\ \underline{\mathbf{-4a} + \mathbf{-2b} + \mathbf{-c} = \mathbf{-23}} \\ 5a + b \quad = 15 \end{array}$$

(By GDC)

Now I have two equations that have two variables in them. If I use elimination again I can single out one variable.

$$\begin{array}{r} 5a + b = 15 \\ \underline{\mathbf{-3a} + \mathbf{-b} = \mathbf{-13}} \\ 2a \quad = 2 \end{array}$$

When I divide both sides by 2, I see that $\boldsymbol{a} = \mathbf{1}$. Now that I have *a*, I can substitute it in to either of the equations with only *a* and *b* in them in order to find *b*. I will do this with the equation $3a + b = 13$.

$3(1) + b = 13$ or $3 + b = 13$

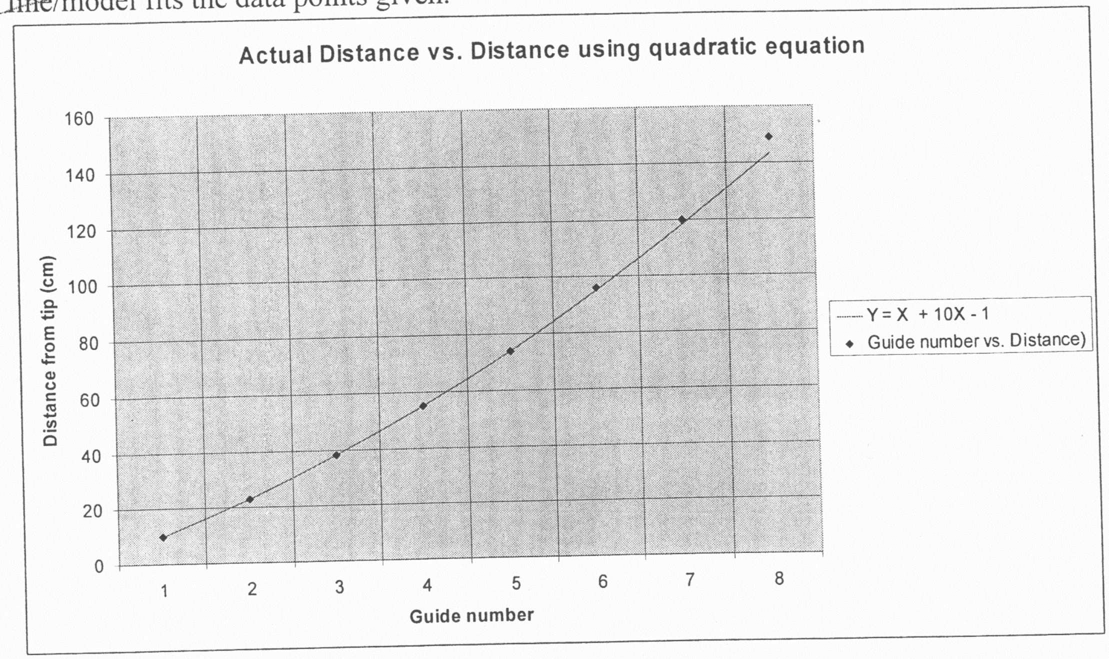
When I subtract both sides by *3* I end up with $\boldsymbol{b} = \mathbf{10}$. Now that I have *a* and *b*, I now can use substitution to place them back into any of the original equations in order to find *c*. I will use the equation $a + b + c = 10$.

$$(1) + (10) + c = 10$$

By subtracting 11 from both sides I can see that $c = -1$. Therefore I can now use substitution and put these numbers back into our original equation and come out with the full equation $Y = X^2 + 10X - 1$.

✓

Now that I have this I can put it onto the graph I have to see if the newly discovered curve ~~line~~/model fits the data points given.



One can see that the curve ~~line~~ is very close to the data provided, but falls short in some places. This is further proven when I substitute *1* through *9* into the equation and end up with the data table:

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip according to Quadratic equation (cm)	10.00	23.00	38.00	55.00	74.00	95.00	118.00	143.00
Actual distance from tip (cm)	10.00	23.00	38.00	55.00	74.00	96.00	120.00	149.00

This proves that the quadratic that was set up very closely resembles the actual data, and is only off on 3 of the guides by a total of 9 cm.

However I plan to make this more accurate, so I will try to find another equation. This time I will set up a cubic with the equation $Y = aX^3 + bx^2 + cX + d$ and I will use the same method of elimination to see what kind of an equation I get. In doing this I will use the four points; (1,10),(2,23),(3,38),(4,55) therefore the equations are:

$$a(1)^3 + b(1)^2 + c(1) + d = 10$$
$$a(2)^3 + b(2)^2 + c(2) + d = 23$$
$$a(3)^3 + b(3)^2 + c(3) + d = 38$$
$$a(4)^3 + b(4)^2 + c(4) + d = 55$$

Or

$$a + b + c + d = 10$$
$$8a + 4b + 2c + d = 23$$
$$27a + 9b + 3c + d = 38$$
$$64a + 16b + 4c + d = 55$$

In order to solve this system, I will first eliminate *d*. I will do this by turning one of the equations negative so that the *d*s cancel out. I will **bold** the changes so that they are easier to follow.

$$\begin{aligned} 64a + 16b + 4c + d &= 55 \\ \mathbf{-27a + -9b + -3c + -d} &= \mathbf{-38} \\ \hline 37a + 7b + c &= 17 \end{aligned}$$

$$\begin{aligned} 8a + 4b + 2c + d &= 23 \\ \mathbf{-a + -b + -c + -d} &= \mathbf{-10} \\ \hline 7a + 3b + c &= 13 \end{aligned}$$

and

$$\begin{aligned} 27a + 9b + 3c + d &= 38 \\ \mathbf{-8a + -4b + -2c + -d} &= \mathbf{-23} \\ \hline 19a + 5b + c &= 15 \end{aligned}$$

(Done with GDC)

From there I will continue to eliminate variables using elimination, next I will eliminate *c* then *b*.

$$\begin{aligned} 37a + 7b + c &= 17 \\ \mathbf{-19a + -5b + -c} &= \mathbf{-15} \\ \hline 18a + 2b &= 2 \end{aligned}$$

and

$$\begin{aligned} 19a + 5b + c &= 15 \\ \mathbf{-7a + -3b + -c} &= \mathbf{-13} \\ \hline 12a + 2b &= 2 \end{aligned}$$

Now I will eliminate the *b*.

$$\begin{aligned} 18a + 2b &= 2 \\ \mathbf{-12a + -2b} &= \mathbf{-2} \\ \hline 6a &= 0 \end{aligned}$$

Therefore $a = 0$. Now I will now use substitution to place this back into the equation $12a + 2b = 2$ in order to solve for *b*.

$0 + 2b = 2$

$b = 1$

Now that I have $b = 1$ and $a = 0$ I can substitute both of these in into $7a + 3b + c = 13$.

$0 + 3 + c = 13$

By subtracting 3 from both sides, I end up with $c = 10$. Now that I have this, I substitute it into $a + b + c + d = 10$ and solve for *d*.

$0 + 1 + 10 + d = 10$

Therefore, by subtracting 11 from both sides, I end up with $d = -1$ which gives me my full equation $Y = X^2 + 10X - 1$. This is the same equation as the quadratic gave us, so it will have the same line fitting to the graph as above.

However I want a line that closer fits the data. So I will try more equations that will hopefully fit closer to the given data as a whole.

For this I will start with a quadratic ($Y = aX^2 + bX + c$) that goes through the points; (1,10),(4,55),(7,120). I will use elimination again to cancel out variables and come to set points.

$$\begin{array}{l} 49a + 7b + c = 120 \\ \mathbf{-16a + -4b + -c = -55} \\ \hline 33a + 3b \quad = 65 \end{array} \qquad \text{and} \qquad \begin{array}{l} 16a + 4b + c = 55 \\ \mathbf{-a + -b + -c = -10} \\ \hline 15a + 3b \quad = 45 \end{array}$$

$$\begin{array}{l} 33a + 3b = 65 \\ \mathbf{-15a + -3b = -45} \\ \hline 18a \quad = 20 \end{array}$$

a = 1.11 (GDC)

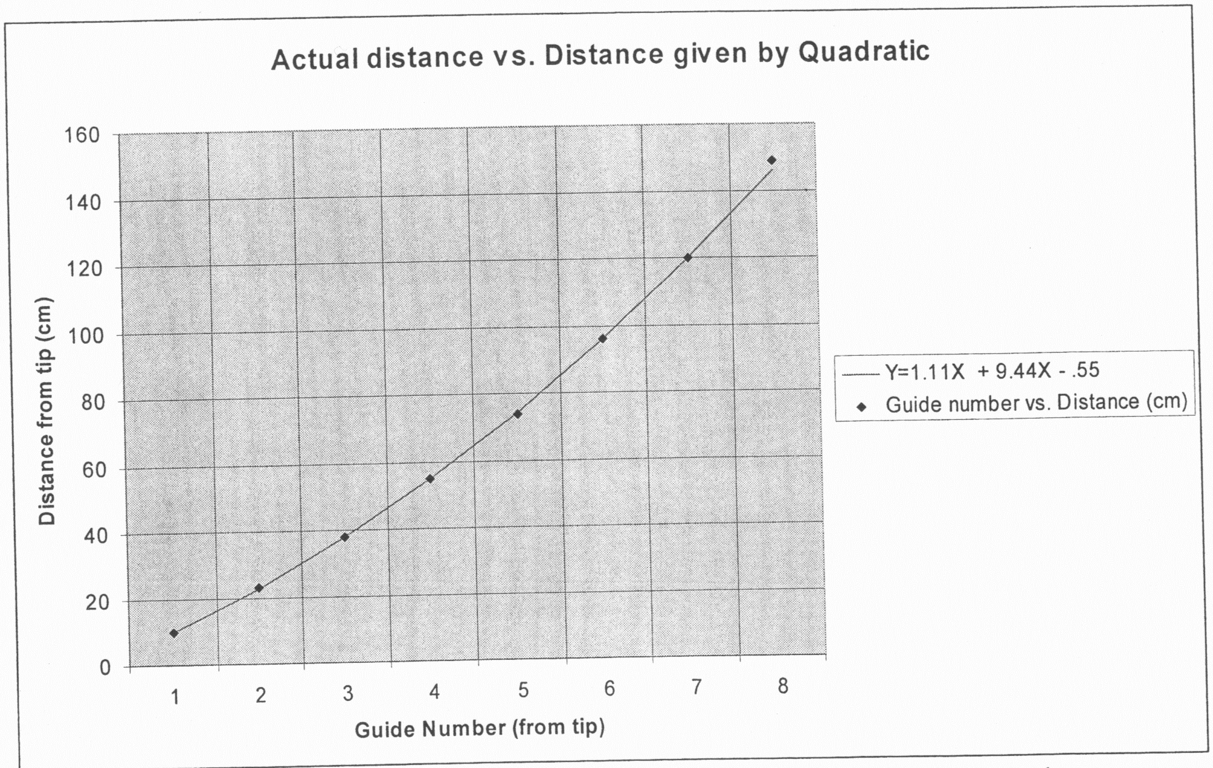
Now if I substitute in each of these into equations above, and use a calculator, I can prove that b and c are as follow:

b = 9.44 (GDC)
c = -.55 (GDC)

Giving the final equation Y= $1.11X^2 + 9.44X - .55$.

Therefore, using a GDC and Excel I come up with the table and graph of;

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip according to Quadratic equation (cm)	10,00	22.77	37.76	54.97	74.4	96.05	119.92	146.01
Actual distance from tip (cm)	10,00	23,00	38,00	55,00	74,00	96.00	120,00	149,00



Here one can see that this quadratic has an even better fit on the given points. It is only off overall by 4.02 cm.

Because I am striving to have an even more exact fit I plan to try another equation. This one will be a cubic ($Y = aX^3 + bX^2 + cX + d$), and will contain the 4 points: (1,10),(3,38),(5,74),(7,120). Again I will use elimination to narrow down the variables and solve for each one and I will continue to **bold** the changes made.

343a + 49b + 7c + d = 120
-125a + -25b + -5c + -d = -74
218a + 24b + 2c = 46

125a + 25b + 5c + d = 74
-27a + -9b + -3c + -d = -38
98a + 16b + 2c = 36

and

27a + 9b + 3c + d = 38
-a + -b + -c + -d = -10
26a + 8b + 2c = 28

(GDC)

Now that I have eliminated the *d* I can move onto eliminating the *c*.

218a + 24b + 2c = 46
-98a + -16b + -2c = -36
120a + 8b = 10

and

98a + 16b + 2c = 36
-26a + -8b + -2c = -28
72a + 8b = 8

(GDC)

Now, I will solve further for *a*.

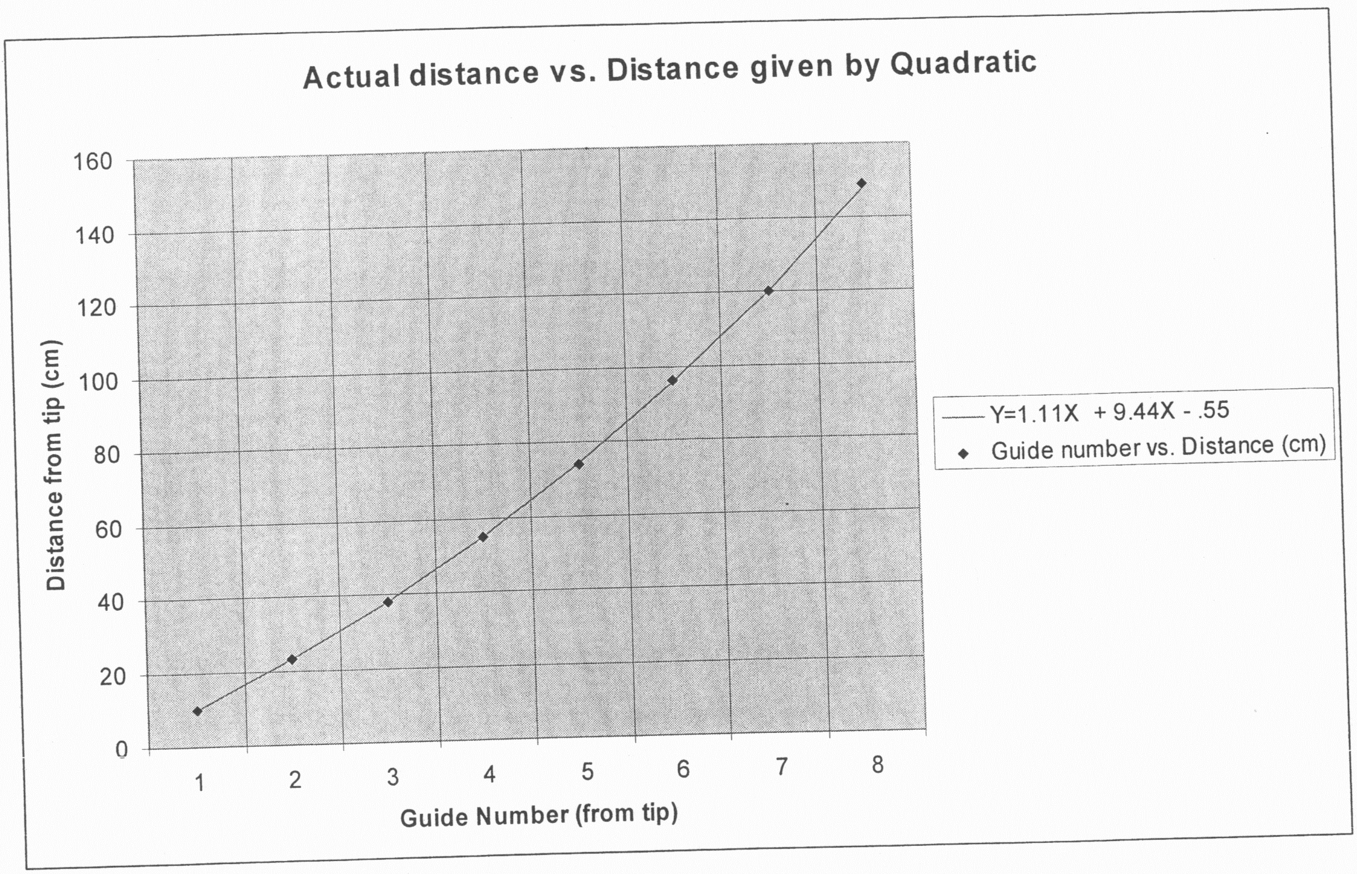
$$\begin{array}{l} 120a + 8b = 10 \\ \underline{\mathbf{-72a + -8b = -8}} \\ 48a \qquad = 2 \end{array}$$

Using a calculator I can solve a to be approximately .042. Now that I have $a = .042$ I can substitute it in, and solve the other by GDC, and I get:

a = .042
b = .625
c = 10.96
d = 1.63

These numbers, when plugged in using substitution, give us the cubic function $Y = .042X^3 + .625X^2 + 10.96X + 1.63$. Which when put into a data table and graph, using a GDC and Excel, looks like this:

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip according to Cubic function (cm)	9.99	23.13	38.00	54.90	74.05	95.70	120.12	147.55
Actual distance from tip (cm)	10.00	23.00	38.00	55.00	74.00	96.00	120.00	149.00



Here it is seen that the graph and table better fit our actual data even more, as overall they only differ from the actual data 2.16 cm.

It seems as though the trend is that if you take more of the actual numbers, from a farther spectrum across the numbers ranging from 1 to 8, you will receive a better fit. Because of this I will make a function using X to the 5th power. The equation will fit the form $Y = aX^5 + bX^4 + cX^3 + dX^2 + eX + f$ I will use the 6 points (1,10),(3,38),(4,55),(5,74),(7,120),(8,149). I will use elimination once again to narrow it down, **bolding** my changes as I go.

$$\begin{array}{l} 32768a + 4096b + 512c + 64d + 8e + f = 149 \\ \underline{\mathbf{-16807a + -2401b + -343c + -49d + -7e + -f = -120}} \\ 15961a + 1695b + 169c + 15d + e = 29 \end{array}$$

$$\begin{array}{l} 16807a + 2401b + 343c + 49d + 7e + f = 120 \\ \underline{\mathbf{-3125a + -625b + -125c + -25d + -5e + -f = -74}} \\ 13682a + 1776b + 218c + 24d + 2e = 46 \end{array}$$

$$\begin{array}{l} 3125a + 625b + 125c + 25d + 5e + f = 74 \\ \underline{\mathbf{-1024a + -256b + -64c + -16d + -4e + -f = -55}} \\ 2101a + 369b + 61c + 9d + e = 19 \end{array}$$

$$\begin{array}{l} 1024a + 256b + 64c + 16d + 4e + f = 55 \\ \underline{\mathbf{-243a + -81b + -27c + -9d + -3e + -f = -38}} \\ 781a + 175b + 37c + 7d + e = 17 \end{array}$$

$$\begin{array}{l} 243a + 81b + -27c + 9d + 3e + f = 38 \\ \underline{\mathbf{-a + -b + -c + -d + -e + -f = -10}} \\ 242a + 80b + 26c + 8d + 2e = 28 \end{array}$$

(GDC)

Now that I have eliminated *f*, I will eliminate *e* using elimination. Here, instead of simply being able to turn the equation negative, I must also multiply the top by 2 in order to make the *e* be eliminated. I will **bold** what I change.

$$\begin{array}{l} \mathbf{31922a + 3390b + 338c + 30d + 2e = 58} \\ \underline{\mathbf{-13682a + -1776b + -218c + -24d + -2e = -46}} \\ 18240a + 1614b + 120c + 6d = 12 \end{array}$$

(GDC)

Here again I must not only make one of the equations negative, but I will also multiply it by 2 so the *e* gets eliminated.

$$\begin{array}{l} 13682a + 1776b + 218c + 24d + 2e = 46 \\ \underline{\mathbf{-4202a + -738b + -122c + -18d + -2e = -38}} \\ 9480a + 1038b + 96c + 6d = 8 \end{array}$$

$$\begin{array}{l} 2101a + 369b + 61c + 9d + e = 19 \\ \underline{\mathbf{-781a + -175b + -37c + -7d + -e = -17}} \\ 1320a + 194b + 24c + 2d \quad = 2 \end{array}$$

$$\begin{array}{l} \mathbf{1562a + 350b + 74c + 14d + 2e = 34} \\ \underline{\mathbf{-242a + -80b + -26c + -8d + -2e = -28}} \\ 1320a + 270b + 48c + 6d \quad = 6 \end{array}$$

(GDC)

Now that I have also eliminated *e* I will eliminate *d*.

$$\begin{array}{l} 18240a + 1614b + 120c + 6d = 12 \\ \underline{\mathbf{-9480a + -1038b + -96c + -6d = -8}} \\ 8760a + 576b + 26c \quad = 4 \end{array}$$

(GDC)

Here I have to multiply *1320a + 194b +24c + 2d = 2* by -3 in order to make the *d* variable opposite of the other *d* variable so that they cancel out.

$$\begin{array}{l} 9480a + 1038b + 96c + 6d = 8 \\ \underline{\mathbf{-3960a + -582b + -72c + -6d = -6}} \\ 5520a + 456b + 24c \quad = 2 \end{array}$$

(GDC)

Here again I must multiply *1320a + 194b + 24c + 2d = 2* by 3 in order to make the *d* variable cancel out.

$$\begin{array}{l} \mathbf{3960a + 582b + 72c + 6d = 6} \\ \underline{\mathbf{-1320a + -270b + -48c + -6d = -6}} \\ 2640a + 312b + 24c \quad = 0 \end{array}$$

(GDC)

Now that the *d* variable is no longer in the equation, I will strive to remove the *c* variable.

$$\begin{array}{l} 8760a + 576b + 26c = 4 \\ 5520a + 456b + 24c = 2 \end{array}$$

The top equation must be multiplied by *24* and the bottom by *-26* so that the *c* variables will cancel each other out.

210240a + 13824b + 624c = 96
-143520a + -11856b + -624 = -52
66720a + 1968b = 44

5520a + 456b + 24c = 2
-2640a + -312b + -24c = 0
2880a + 144b = 2

(GDC)

Now there are just 2 variables left, I will solve to remove the *b* variable.

66720a + 1968b = 44
2880a + 144b = 2

The top equation must be multiplied by *144* and the bottom by *-1968* as to reach a common value so that *b* can be eliminated.

9607680a + 283392b = 6336
-5667840a + -283392b = -3936
3939840a = 2400

(GDC)

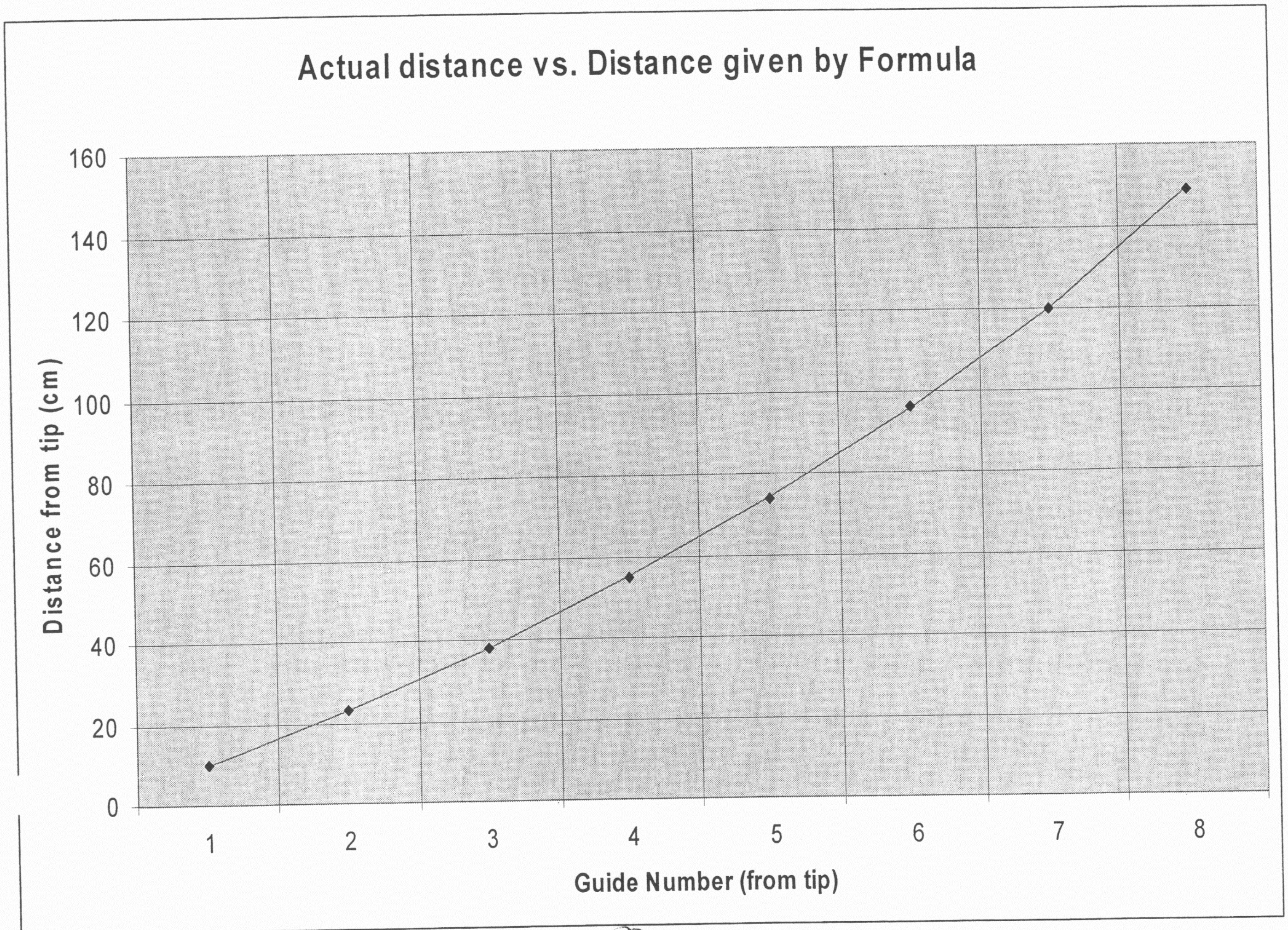
Now using a calculator, I can determine that $a = 6.09x10^{-4}$. If I use substitution to put this in elsewhere I get the following numbers for my variables:

$b = 1.71x10^{-3}$
$c = -8.92x10^{-2}$
d = 1.50
e = 9.04
f = -.41

Therefore, the complete formula is $6.09x10^{-4}X^5 + 1.71x10^{-3}X^4 - 8.92x10^{-2}X^3 + 1.50X^2 + 9.04X - .41$

The table that is drawn and the graph fit near perfectly to the information given as shown:

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip according to function to the fifth power(cm)	10.04	23.00	38.09	55.10	74.11	95.51	120.12	149.20
Actual distance from tip (cm)	10.00	23.00	38.00	55.00	74.00	96.00	120.00	149.00



These tests prove that this is the best fit line. Overall, it is off by 1.15 cm. The reason for this is that more of the points are actually part of trying to figure out the equation for the best fit line. The differences from the predicted distance from the tip, and the actual distance from the tip are little to none with this model. This model fits the data almost perfectly, so it should be useful for figuring out where another guide would go on a fishing rod of the same length.

In order to figure out where the 9th guide would go on such a fishing rod, I will substitute in *9* for X in my best fit Quadratic model so that:

$$1.11(\mathbf{9})^2 + 9.44(\mathbf{9}) - .55 = 174.32$$

However, if I was looking to find a more accurate fit, I would substitute *9* into the best fit equation that I came up with:

$$6.09\times10^{-4}(\mathbf{9})^5 + 1.71\times10^{-3}(\mathbf{9})^4 - 8.92\times10^{-2}(\mathbf{9})^3 + 1.50(\mathbf{9})^2 + 9.04(\mathbf{9}) - .41 = 184.60$$

(By GDC)

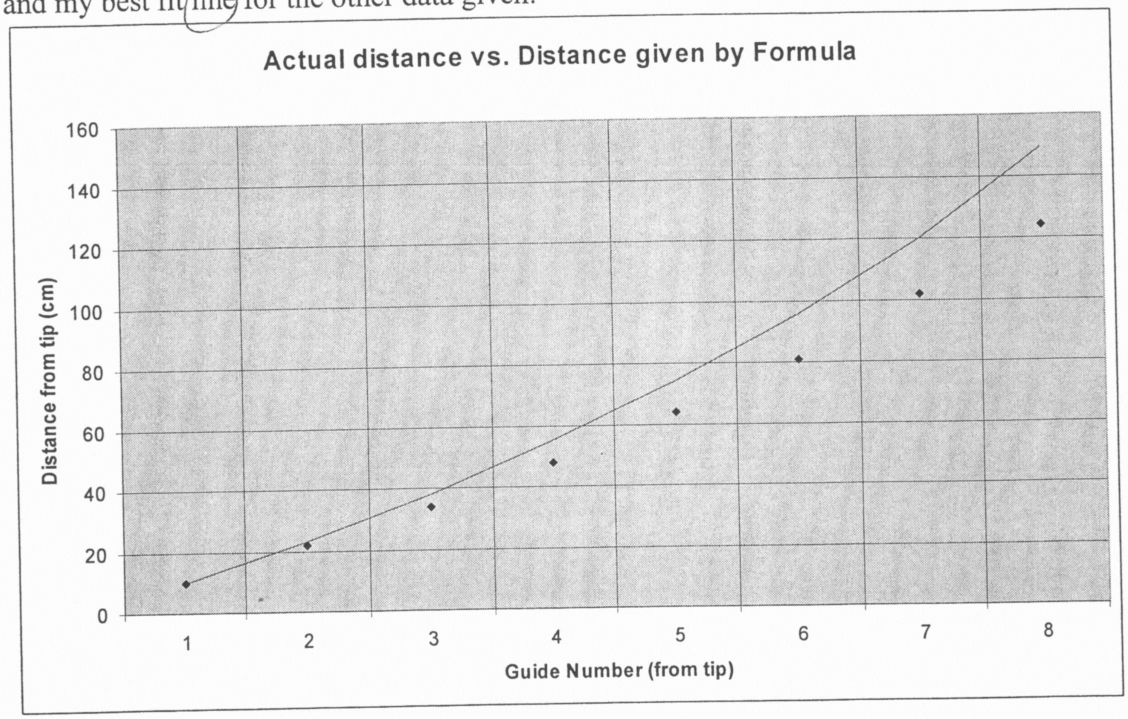
If one were to add a 9th guide here the rod could be easier to cast. It also could slow down the casting because it would be so close to the reel. It would also keep the line closer to the rod at all times so that it does not ever get tangled up. The 9th guide would not be too

close that it would ruin how the rod works. If the guides at much friction it may slow down the casting rate which could lead to less efficient fishing. The cost of adding another guide would be minimal, so that would not be a large disadvantage.

If another person was to have a fishing rod with a length of 300 cm. the table would look like this:

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10.00	22.00	34.00	48.00	64.00	81.00	102.00	124.00

This data does not fit my model very well; the graph below shows the data points given, and my best fit line for the other data given:



The line does not fit the data very well. Many changes would need to be made in mine. The new data is spread out further and has a steadier slope rather than the steeper slope that the other data had.

The model I made only works for a rod of that size and with guides that are placed in the same pattern on the rod. If it is a different brand of fishing rod, or a different genre my model will probably not fit. The data collected was from a certain fishing rod, not all fishing rods share the same build. A good way to test all of this would be to find the most efficient fishing rod and figure out a formula that best fits this rod and then distribute it to others.

Through multiple different tests I learned that the best fit model is the one that when made uses the most known points and the widest variety of known points. The best fit model I chose was to the fifth power because it used 6 different points that I knew were correct in making the formula.